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PHYSICS OF TECHNOLOGY

COORDINATED BY AMERICAN INSTITUTE OF PHYSICS

THE PILE DRIVER

Kinematics, Work, and Energy Transformations

THE PILE DRIVER

A Module on Kinematics, Work, and Energy Transformations

FVCC

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MCGRAW-HILL BOOK COMPANY

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The Physics of Technology modules were produced by the Tech Physics Project, which was funded by grants from the National Science Foundation. The work was coordinated by the American Institute of Physics. In the early planning stages, the Tech Physics Project received a small grant for exploratory work from the Exxon Educational Foundation.

The modules were coordinated, edited, and printed copy produced by the staff at Indiana State University at Terre Haute. The staff involved in the project included:

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This module was written and tested at Florissant Valley Community College.

This module was produced through the efforts of a number of people in addition to the principal authors. Laboratory experiments were developed with the assistance of Donald Mowery. Illustrations were prepared by Robert Day and Susan Snider. John Yoder, III, coordinated art work, and he also prepared the Instructor's Manual. Reviews were provided by members of the Physics of Technology Steering Committee and participants in the NSF Chautauqua Program. Finally, Wendy Chytka typed the various module drafts, coordinated preliminary publication efforts, and acted as a liaison person with the Terre Haute Production Center where final copy was being produced. To all these persons, we are indebted.

Cover photograph taken by Dennis Purdy.

The Pile Driver

ISBN 0-07-001719-0

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GOALS FOR SECTION A

The following goals state what you should be able to do *after* you have completed this section of the module. These goals should be studied carefully as you go through the module and as you prepare for the post-test. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal.

1. *Goal:* Know how the "set" of a pile, with a given pile driver, changes with the pile's penetration into uniform ground.

   Item: At one time, in driving a pile into uniform ground, the set produced is 20 cm, while at a different time it is 6 cm. Which occurred earlier?

2. *Goal:* Know the general effects of hammer weight and free-fall distance on the set of a pile.

   Item: One pile driver produces a set of 8 cm at a penetration of 1.5 m while a second produces an 11-cm set with the same type pile at a 2-m penetration in the same ground. Which of the following statements could be true?

   a. The first pile driver has a heavier hammer and greater free-fall distance.

   b. The first pile driver has a heavier hammer and same free-fall distance.

   c. The first pile driver has a lighter hammer and same free-fall distance.

   d. The first pile driver has the same hammer weight and a greater free-fall distance.

3. *Goal:* Understand in a general way how the speed of a freely falling object changes with time.

   Item: Place the following speeds of a freely falling object in the proper time order, earliest first and latest last: 5 m/s, 1.5 m/s, 3 m/s, 2 m/s.

4. *Goal:* Understand qualitatively how the speed of free fall is related to weight and air resistance.

   Item: A three-gram leaf, a three-gram penny, and a two-gram dime are dropped simultaneously from the same height. In what order will they strike the floor?

Answers to Items Accompanying Previous Goals

1. The 20-cm set occurred earlier at less penetration.

2. c.

3. 1.5 m/s, 2 m/s, 3 m/s, 5 m/s.

4. Dime and penny simultaneously and before leaf.

SECTION A

EARLY PILE FOUNDATIONS AND PILE DRIVERS

The idea of pushing long poles, called *piles,* into soft ground in order to support structures above the ground is very old. Five thousand years ago, stone age people on the shores of Swiss lakes drove piles into the muddy ground to support the floor joists of their wooden huts. Figure 1 shows the piles and flooring of such a hut, and its final appearance.

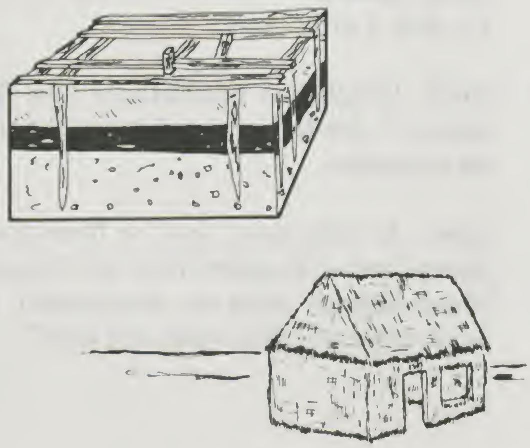


Figure 1.

You might wonder why anyone would go to so much trouble to build on soft ground, particularly in an age when the world was not very crowded. Perhaps it was because the people obtained their food from the lakes and streams and wished to be as close to them as possible. Perhaps the muddy terrain made these people more secure from attack. We do know that the latter reason was an important one for the building of entire cities on marshy ground during the Roman Empire. Ravenna, in northern Italy, which was built in marshes, became a major Roman stronghold against attacks by invading tribes. Later, about 1000 A.D., the city of Venice flourished as one of the last outposts of the Eastern Roman Empire. Venice was built on marshes and islands with the extensive use of wooden piles, and it never needed a city wall for protection. As late as the seventeenth century, one large church in that city was under construction for more than fifty years. This long building time was needed, at least partly, because it took decades to drive the necessary piling.

Any device which forces a pile into the ground is called a *pile driver.* The Romans used a simple pile driver which consisted of a wooden frame supporting a heavy stone hammer on a rope. The hammer could be lifted by a gang of men or work animals, then released to fall on the top, or *head,* of the pile.

CURRENT USE OF PILES

In our own times, pile foundations are sometimes necessary in ground that you would not consider very soft if you walked over it. Heavy, multi-story buildings might settle in undesirable ways without the use of piles.

There are four different ways in which piles are effective. First, piles can transfer a load through a soft layer of soil to a hard layer of rock below (Figure 2A). In this case, most of the load is supported by the ends of the piles. Secondly, piles can transfer a load through a soft layer of soil to a more dense layer. The friction of forces exerted by the lower layer on the lower part of the pile helps to support the load (Figure 2B). Thirdly, piles can be supported by friction along their entire lengths (Figure 2C). Finally, piles are used as soil *compactors,* making a loose soil more able to withstand the load of a building (Figure 2D). In this case, the load is not necessarily applied to the pile but rather to the compacted soil around the pile.

Piles are also used as protective bumpers. For example, piles are often used to protect bridge supports on rivers from collisions with barges or other river traffic.

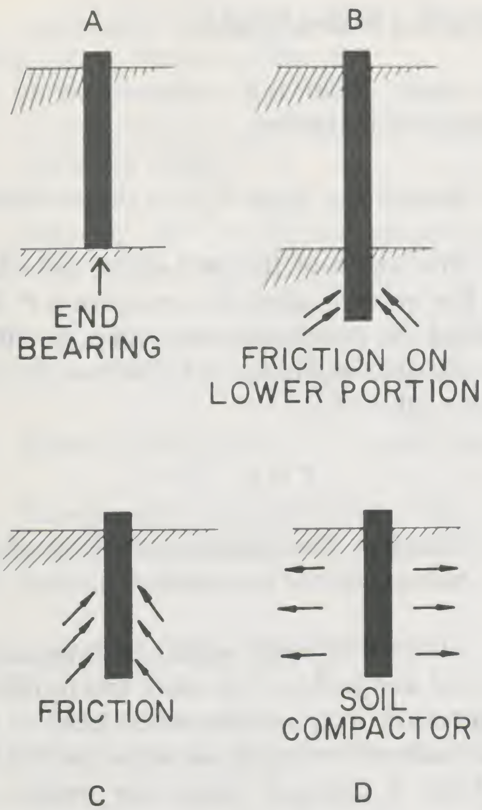


Figure 2.

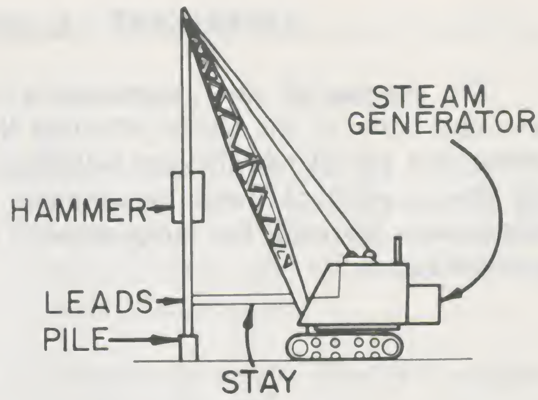


Figure 3.

MODERN PILE DRIVERS

Many modern pile drivers still use the basic principles of the ancient Roman pile driver: a large weight is lifted then dropped on the head of the pile. Today machines, rather than men or animals, lift the hammer. A typical driving rig is shown in Figure 3. Two parallel steel channels, called *leads,* guide the hammer as it moves. A *stay* holds the leads in the proper position.

The simplest type of pile driver is the *drop hammer,* which uses a 250- to 1000-kg steel hammer, raised 2 or 3 m by a winch, then dropped repeatedly on a pile. This type is relatively slow because of the time required to raise the hammer with the winch each time. Drop hammers are most like the ancient types.

Steam hammers use steam in a cylinder to lift the hammer, which is called a "ram." A *single-acting* steam hammer lifts a heavy ram and allows it to drop a distance of about 1 m before striking the pile. It can produce a uniform series of blows at a rate of nearly one a second. A *double-acting* steam hammer uses steam to raise the hammer and again to drive the ram down. This type can deliver about two blows per second. The double-acting hammer is faster and each blow is more forceful, but usually the blows are not as uniform as those of the single-acting steam hammer. Figure 4 illustrates both types of steam hammer.

Ancient pile drivers, the modern drop hammer, and the single-acting steam hammer all use gravity acting on a lifted weight. These types use the principles of physics we wish to study in this module. You can gain some understanding of their operation by doing Experiment A-1.

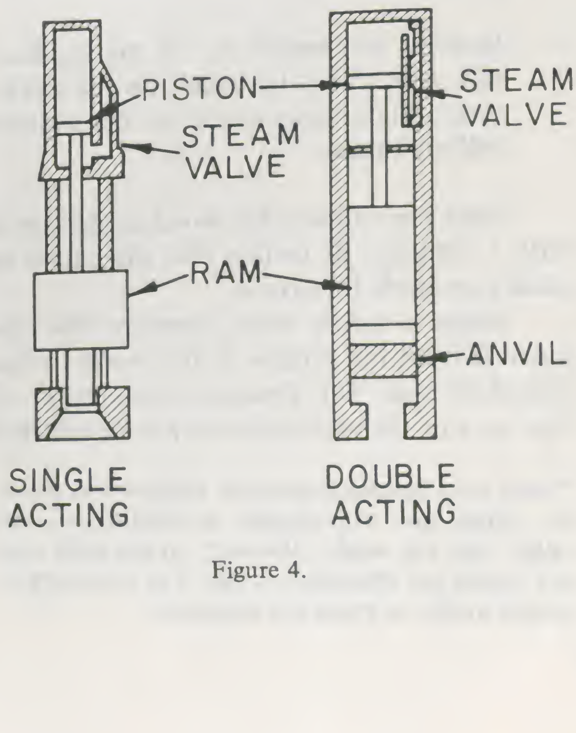


Figure 4.

EXPERIMENT A-1. Driving a Pile with a Falling Weight

The purpose of this experiment is to determine some of the factors affecting the driving of a pile. To do this you will drive a nail into a piece of wood by dropping a weight onto the nail. The setup required is shown in Figure 5.

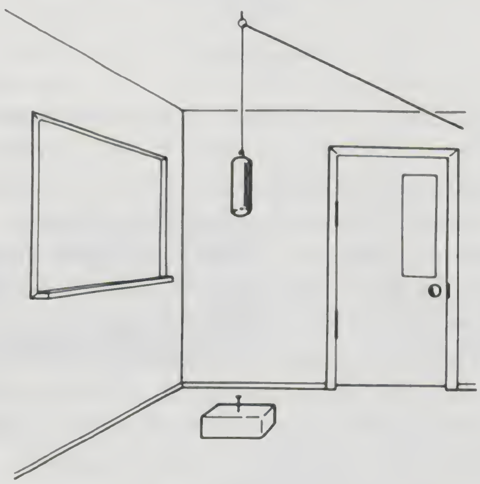


Figure 5.

1. Weigh* each of the three nails, the small weight, and the large weight and record the results on the worksheet in the back of the module.

2. Measure the length L_1 of the smallest nail, and record the result on the worksheet. State your result to the nearest half-millimeter.

Start this nail into the wood by driving it with a hammer. Be certain that the nail is as close as possible to vertical.

Using a metric scale, measure the distance D from the surface of the wood to the top of the nail. This distance is the length of that part of the nail which has *not* penetrated the wood. Make this measurement to the nearest half-millimeter.

3. Record the value of D in the worksheet.

The length of that part of the nail which is in the wood is called the *penetration P.* You can find the penetration by taking the difference of nail length L_1 and distance D. Thus for any value of D,

$$P = L_1 - D$$

4. Calculate the initial penetration of the nail and record the result in Table 1.

Attach the small weight to one end of the cord and pull on the other end to lift the weight until it is just above the head of the nail. Position the block of wood so that the nail-head is centered below the weight. By pulling on the cord, lift the weight to a height of 200 cm above the top of the nail. Use a meter stick to measure this height.

CAUTION: Before dropping the weight, make sure that no one is near the nail. The weight may bounce after hitting the nail.

Drop the weight on the nail.

5. Measure the length D of the nail above the wood and record the result in column one of Table 1.

The *set* is the amount by which the nail is driven with each blow. You can find the set for this blow by taking the difference in the two values of D you have found so far.

6. Calculate the value of the set and record the result in column two of Table 1.

Calculate the value of the penetration of the nail now.

7. Record the new value of penetration in Table 1.

*Since your balance is probably calibrated in grams, not dynes, you will actually be finding the *mass* rather than the weight. However, on the earth mass and weight are proportional, and it is customary to specify weights in grams and kilograms.

Again drop the weight from a height of 200 cm above the top of the nail.

8. Measure and record the length of the nail above the wood.

9. Calculate and record the set and new value of penetration.

10. Repeat the above process until a penetration of 8 cm or about ½ of L is achieved.

11. Measure the length L_2 of another nail.

Repeat these measurements, this time dropping the small weight from a height of 100 cm.

12. Record your data in Table 2.

Measure the length L_3 of a third nail and repeat the measurements, this time dropping the large weight from a height of 100 cm.

13. Record your data in Table 3.

14. Plot a graph of set on the vertical axis and penetration on the horizontal axis for each of the three tables of data. All three graphs may be plotted on one sheet of graph paper.

15. Examine the graphs. What factors appear to affect the set?

16. What happens to the set as penetration increases?

17. How does the height from which the weight falls affect the set?

18. How does the mass of the falling weight affect the set?

SET DEPENDS ON PENETRATION

The distance by which the pile advances as a result of a single blow is called the *set.* In Experiment A-1, you saw that the set is not a constant number, even if the hammer weight and free-fall distance are always the same. Instead, the set is relatively large with the first few blows, and finally decreases to a fairly constant value. Figure 6 shows the relationship of set to penetration for two different hammer sizes. This is a reasonable result in a uniform "ground," like the wood you are using. Each blow is the same but the resistance of movement increases as more of the pile enters the ground: more area of the pile surface is exposed to friction.

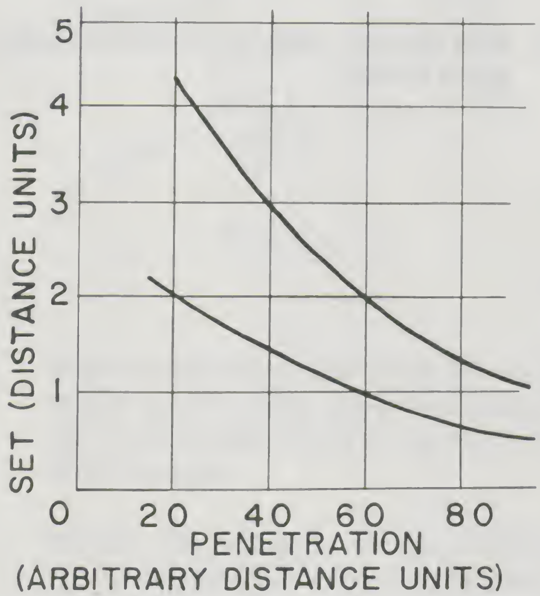


Figure 6.

Question 1. How do you think the set would change with the penetration in the end-bearing use of piles, shown in Figure 2?

SET DEPENDS ON PILE DRIVER CHARACTERISTICS

In Experiment A-1, you discovered that the set is larger for larger hammer weights at any given penetration. Also, the set is larger for a longer free fall of the hammer. The results are in agreement with common-sense expectations. You would expect a heavier weight or a longer fall to produce a more forceful blow and drive the pile farther.

Why is it true that a longer free fall produces a more forceful blow? Or, to put the question another way, how does the pile "know" the hammer has fallen a longer or shorter distance? At the instant it hits the pile, the hammer must have some property which depends on the height of free fall and which affects the driving force on the pile. From your observations in Experiment A-1 and in everyday life, you may have noted that the farther an object falls, the faster it appears to move. Thus, the speed of the falling hammer is a likely candidate for a property that affects the driving force in some way.

Question 2. The graphs in Figure 7 show the set versus number of blows for two drop hammers with the same hammer weights working on the same type piles in the same soil. What is the difference in the two pile drivers?

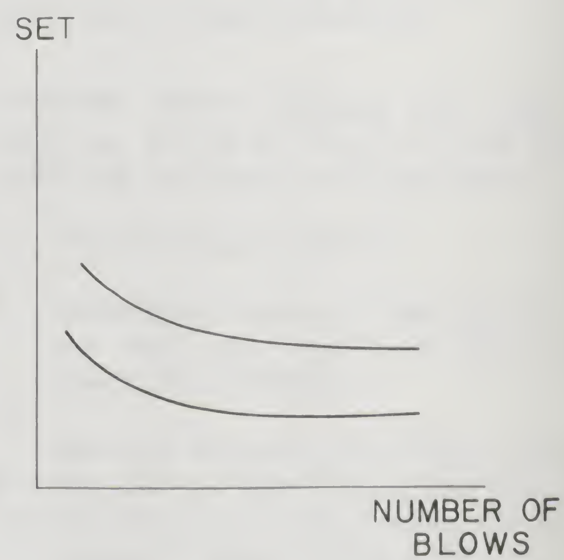


Figure 7.

SPEED AND WEIGHT

When we consider the effects of the hammer weight, the question naturally arises as to whether they are due to the weight in itself or due to a greater speed at impact of

the heavier weight. Does a heavier weight fall faster than a lighter one?

You can answer that question for yourself by releasing two spheres of the same size but different weight simultaneously. For example, a steel ball and a rubber ball, each about the same diameter, will both strike the floor at the same time, as closely as your senses can detect. This result indicates that both objects have the same speed at the floor, and indeed all through the trip, for if either sphere moved faster for any length of time, it would arrive at the floor sooner.

There is a story that Galileo Galilei (1564-1642) actually tried this very experiment from the leaning tower of Pisa. No one knows for sure whether the story is true, but it is certain that Galileo knew that two different weights fall with the same speed.

AIR RESISTANCE

Of course, you have also made many observations in which different objects did not fall with the same speed: a feather does not fall as fast as a brick, nor a parachutist as fast as a person without a parachute. In such cases, *air resistance* plays an appreciable part. If the air resistance were removed, a feather would fall as fast as a brick, and sky diving would be an unpopular sport. Perhaps you have seen the demonstration of this fact performed by an astronaut on the moon, where there is no air. He dropped a feather and a wrench simultaneously and watched them strike the lunar surface together. For compact masses falling short distances, air resistance is usually negligible. Certainly, with the pile driver we shall not have to worry about air resistance.

OPTIMUM HAMMER WEIGHT AND FREE-FALL DISTANCE

From these observations, we conclude that the *weight* of the hammer also has a significant effect on the driving of the pile: a heavy hammer produces a larger set than a light hammer striking at the same speed. We also know that a longer free-fall distance produces a larger set. Why then don't pile drivers use an extremely long free fall and an extremely heavy weight to drive the pile in one or two blows?

One practical reason for not using extreme values of hammer weight or free-fall distance is that there is the danger of damaging the pile so that it can no longer support a load. The situation in which the blows are so forceful that the pile fails (for example, by bending or splitting) is called *over-driving.* Experience has shown that the best results are usually obtained with a hammer that is about twice as heavy as the pile and that falls a few meters.

SUMMARY

You have seen that the set of a pile being driven in uniform "soil" depends on a number of factors. The set decreases with the number of blows because the frictional resistance increases as the penetration increases.

Also, you demonstrated that the set increases with increasing hammer weight or increasing free-fall distance, each of which produces a more forceful blow. Although the longer fall may produce a more forceful blow because it causes a higher hammer speed at impact, the extra forcefulness of a heavier hammer does not come from a higher speed. Hammers of different weights dropped from the same height have the same speed at impact.

Practical considerations, such as the avoidance of over-driving (driving the pile to failure), limit the hammer weight to several times the pile weight and the free-fall distance to a few meters.

In general, the heavier the weight of a falling object the greater the force with which it hits something. Also, the farther it falls, the greater its speed and the greater the force. For the same distance of fall, compact objects have the same speed regardless of their weights.

GOALS FOR SECTION B

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal.

1. *Goal:* Know the definitions of average speed, instantaneous speed, and acceleration for straight-line motion.

   Item: An automobile changes its speed from 25 m/s (56 mph) to 37 m/s (83 mph) in 8 s. What is its acceleration during this time?

2. *Goal:* Know the relation between distance traveled, initial speed, acceleration, and time for a constant acceleration straight-line motion.

   Item: A modified stock-car drag racer covers 400 m in 9 s from a standing start. What is its acceleration?

3. *Goal:* Know the direct relation between final speed, acceleration, and distance traveled for a straight-line constant acceleration where the initial speed is zero.

   Item: If the acceleration of a bus is 1.5 m/s^2, how far does it travel in reaching 15 m/s from a stop?

4. *Goal:* Be able to work free-fall problems on the earth's surface.

   Item: A rock is thrown directly downward off the rim of the Grand Canyon at an initial speed of 10 m/s. How far does it travel in 5 s?

Answers to Items Accompanying Previous Goals

1. 1.5 m/s^2
2. 9.9 m/s^2
3. 75 m
4. 173 m

SECTION B

DEFINITION OF AVERAGE SPEED

In the last section we mentioned the speed of the hammer when it strikes the pile as one possible factor in determining the set. We also noted the common observation that falling objects seem to speed up as they fall. This speed increase is particularly noticeable in the pile driver experiment you performed, because the hammer always starts from rest (with zero speed). To investigate how a falling object, such as the pile driver hammer, gains speed, we shall need precise definitions of the quantities we are discussing so that we can know how to measure them.

One of the simplest quantities describing the motion of an object is its *average speed.* The average speed v_{av} is defined as the distance d traveled divided by the time t required for the object to travel that distance. In equation form

$$v_{av} = d/t \qquad (1)$$

(Definition of Average Speed)

The units of speed are distance units over time units, such as meters/second (m/s).

Typical American automobile speedometers are calibrated in units of miles per hour (mi/h), while cars in most of the rest of the world are in units of kilometers per hour (km/h). In scientific measurements throughout the world, speed is usually recorded in m/s.

Example Problem. A ball falls 4 m in 0.8 s. What is its average speed during the fall?

Solution. You are given both $d = 4$ m and $t = 0.8$ s. Dividing distance by time gives average speed v_{av};

$$v_{av} = d/t = 4 \text{ m}/0.8 \text{ s}$$

$$v_{av} = 5 \text{ m/s}$$

Problem 1. A 500-kg pile hammer falls 3 m in 0.9 s. What is its average speed during the fall?

Problem 2. At an average speed of 20 m/s, how long would it take you to drive 100 km?

INSTANTANEOUS SPEED

Although it represents an important first step in our knowledge, the average speed is not a very complete description of the motion of a falling weight. At the very start of the fall of a pile driver hammer, it is not moving at all: it has zero speed. At the end of its fall it is moving most rapidly. We can think of the hammer as having a different speed at every instant of its downward trip. The average speed represents some kind of typical value. The speed at a particular instant is known as the *instantaneous speed.* It is still related to a distance divided by a time, but not quite so simply as average speed.

The concept can be illustrated in considering the fall of a drop hammer through 4.9 m. This actually takes 1 s. From Equation (1), you can calculate the average speed. The result is 4.9 m/s.

However, as shown in Figure 8, over the last half of the trip (2.45 m) the average speed will be larger. This part of the fall actually takes only about 0.29 s. The average speed for this half of the trip is therefore about 8.4 m/s.

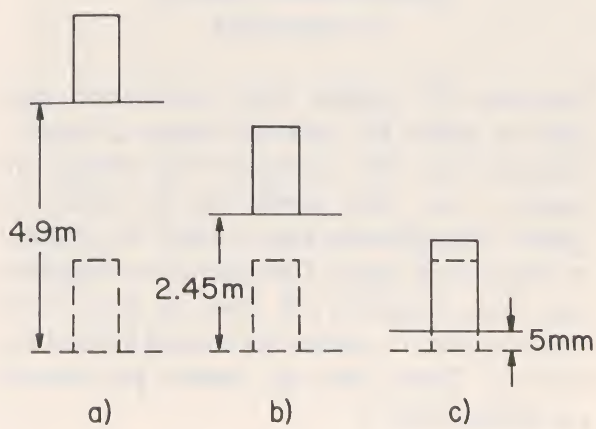


Figure 8.

Proceeding in this manner, you could conceivably measure only the last 1/1000 of the fall (the last 5 mm), dividing that number by the small fraction of a second it takes the weight to traverse that distance. You would arrive at a value for average speed near 10 m/s, which could reasonably be used as the instantaneous speed of the hammer at the end of its fall.

In other words, the instantaneous speed at any instant can be thought of as the average speed over a very short time interval containing that instant. In an automobile, the speedometer reads instantaneous speed. As you know, the average speed over a long trip is usually much less than the highest instantaneous speed on the highway.

During an interval of time when the speed of a moving object doesn't change, the average speed and the instantaneous speed are the same. However, your observations of the falling hammer should have indicated that there is no significant period of time when the speed of a falling object is constant.

DEFINITION OF ACCELERATION

Whenever the speed of an object changes, the object is said to be *accelerating* or to have an *acceleration*. In the case of an object moving in a straight line, such as the pile driver hammer or any other dropped object, the average acceleration a_{av} is defined in terms of the initial speed v_0, the final speed v, and the time t required for the speed to change from its initial to its final value.

$$a_{av} = (v - v_0)/t \qquad (2)$$

(Definition of Average Acceleration)

Equation (2) implies that acceleration can also be called the rate of change of speed, because you first calculate the change in speed $v - v_0$, then divide by the time required. This gives the rate at which the change in speed took place. The units of acceleration are, from Equation (2), units of speed over units of time, or meters per second divided by seconds. These units are meters per second per second (m/s^2).

Example Problem. If an automobile increases its speed from 10 m/s to 24 m/s in 12 s, what is its average acceleration during that time?

Solution. Given are $v_0 = 10$ m/s, $v = 24$ m/s, and $t = 12$ s. These can be substituted directly in Equation (2) to give

$$a_{av} = (v - v_0)/t$$

$$= [(24 - 10)\ \text{m/s}]/12\ \text{s}$$

$$a_{av} = (14\ \text{m/s})/12\ \text{s} = 1.2\ \text{m/s}^2$$

This result means that during the 12 s in question, the automobile is increasing its speed by 1.2 m/s every second.

Question 3. What does a negative average acceleration mean?

Problem 3. An automobile starting from rest can reach 20 m/s in 8 s. What is its average acceleration?

Problem 4. A pile driver hammer hits a pile at 10 m/s and comes to rest in 0.15 s. What is its average acceleration during this time?

Problem 5. A drag-race car can accelerate at 6 m/s^2. How long does it take to reach a speed of 30 m/s from a standing start?

CONSTANT ACCELERATION

It is possible for an object in motion to have an acceleration that changes as it moves. In this case, we could think of the instantaneous acceleration as the acceleration over a very short time interval. But if the acceleration is constant, the average acceleration is the same as the instantaneous acceleration, just as average speed is the same as instantaneous speed when the speed is uniform.

From your observations on the pile driver, you probably cannot tell whether a freely falling hammer has a constant acceleration or a varying acceleration, although it is clear that the hammer is accelerated.

One way to investigate this question is to

measure the distance a falling object travels in each of a succession of equal short time intervals. For sufficiently short time intervals we can consider the average speed over the interval as the instantaneous speed. Because of the object's acceleration, we expect the calculated speeds to increase as the object falls farther. If the acceleration is constant, Equation (2) implies that the change in speed from one interval to the next is constant. Otherwise, the change in speed between successive intervals is different.

You can use this method to investigate a freely falling object by doing Experiment B-1.

EXPERIMENT B-1. Motion of a Freely Falling Body

The purpose of this experiment is to determine the motion of an object as it falls without restraint. For this experiment you will need some way to record the position of the object at successive time intervals. You may use a spark timer, a ticker-tape timer, a stroboscope and Polaroid camera, photoelectric gates, or any other device for measuring short time intervals that may be available in your laboratory. Whatever way the data are taken, you should end up with a "picture" of the position of the falling object versus time for equal time intervals, as shown in Figure 9. You will also need to know the value of the time interval if your answers are to be in seconds.

Get a set of data on a tape or photograph like that of Figure 9. A steel ball is a good object to use for a falling body.

Figure 9. The positions of a falling object in equal time intervals.

1. Remember that the marks are made at equal time intervals. As the object falls, does its speed change?

2. Does the speed of the object increase or decrease as it falls?

We now wish to examine quantitatively the changes in speed as the object falls.

3. Starting with the first clearly defined mark, measure each of the distances Δd* that the object fell in each of the first ten time intervals. (If you used a spark timer or a ticker-tape timer, the dots may be too close together to do this conveniently. In that case use every second dot or every fifth dot, or whatever conveniently gives at least ten intervals. Remember to multiply the basic time unit by two or five if you do this. With a strobe photo it is convenient to punch holes in the picture at the various positions and project this onto a chalkboard with an overhead projector. If you are clever, you can project it "life size" and get your measurements accordingly.) Record these distances in Table 4 in the worksheets.

4. Measure and record the distance d fallen from the starting point to each mark.

5. Calculate and record the average speed v_{av} for each interval from the equation:

$$v_{av} = \Delta d / \Delta t$$

Here Δt is used to signify the time interval between two measured positions. From Figure 9,

*The Greek letter *delta* (Δ) is often used to show a difference or a change in two different values of a quantity. Here, in the third time interval, for example, $\Delta d = d_3 - d_2$.

$$\Delta t = t_1 - 0$$

$$\Delta t = t_2 - t_1$$

$$= t_3 - t_2, \text{ etc.}$$

6. From the definition, $a_{av} = (v - v_0)/\Delta t = \Delta v/\Delta t$, find the average acceleration. You can do this by calculating the difference of the speeds in one interval and the next one, and dividing this difference by the time between intervals; this is also Δt. Enter your value of average acceleration between each pair of time intervals into Table 4.

7. Does the acceleration vary as the object falls, or is it nearly constant?

8. What is the average of these ten values of the average acceleration?

9. To determine how the speed changes with the distance the object has fallen, plot a graph of v_{av} versus d. Put v_{av} on the vertical axis and d on the horizontal axis.

10. Is this graph a straight line?

11. If the graph cannot possibly be a straight line, try plotting different relationships between v_{av} and d, looking for a straight-line graph. For example, try plotting v_{av}^2 versus d.

12. Is this graph nearly a straight line?

13. If the graph is nearly a straight line, draw the straight line which best fits your data points. Find the slope of that line. (Slope is the rise divided by the run between two points. If you don't know how to do this, get some help from your teacher.)

14. What are the units of the slope of the line in 13?

15. How does the slope of the line compare with the average acceleration you found in step 8?

ACCELERATION DUE TO GRAVITY

The effect that causes objects such as the hammer of a pile driver to accelerate toward the center of the earth is called *gravity*. In Experiment B-1 you found that the acceleration due to gravity at the earth's surface is a constant and equal to about 10 m/s^2. (A more precise value is 9.8 m/s^2.) This result means that whenever you drop something near the earth's surface it gains a downward speed of 9.8 m/s every second: at the end of the first second it is traveling 9.8 m/s, at the end of the second second it is traveling 19.6 m/s, etc. Your earlier observations showed that different weights, when dropped from the same height at the same time, reach the floor at the same time. Further, they have the same speed at any instant, as long as air resistance is neglected. We therefore conclude that all objects have this same acceleration when dropped. An object accelerated only by gravity is said to be in *free fall.*

We can combine the experimental results with the mathematical definitions of this section to obtain more general results for objects in free fall. For one thing, we do not have to confine ourselves to objects which are dropped, since Equation (2) provides for an initial speed. The initial speed can be directed up or down, which requires us to be careful about the signs (+ or −) of the quantities we use. A negative acceleration may indicate an acceleration opposite in direction to the initial speed. When it results in a slowing down of the object, a negative acceleration is called *deceleration.* A more general sign choice that is often used assigns one sign to the up-direction and the opposite sign to the down-direction. Which convention you use does not make any difference in the physical meaning of the final result as long as you are consistent. We shall adopt the plus sign (+) for up and the minus sign (−) for down. The symbol g is usually used for the acceleration of gravity at the earth's surface, and the convention we have adopted indicates that its numerical value should always be entered into our equations as a negative number ($g = -9.8\ m/s^2$). Thus for free fall, from Equation (2),

$$-9.8\ m/s^2 = (v - v_0)/t$$

Solving this for the final speed:

$$v = v_0 - (9.8\ m/s^2)t \qquad (3)$$

Example Problem. A ball is thrown upward with an initial speed of 50 m/s. How fast is it moving 8 s later?

Solution. You are given $v_0 = +50$ m/s (+ for up), and $t = 8$ s. Both can be substituted in Equation (3):

$$v = v_0 - (9.8\ m/s^2)t$$

$$= 50\ m/s - (9.8\ m/s^2 \times 8\ s)$$

$$v = 50\ m/s - 78.4\ m/s$$

$$v = -28.4\ m/s$$

The answer is 28.4 m/s *downward* (that's what the minus sign means). After 8 s, the ball has gone up, turned around, and is coming back down.

Example Problem. If a ball is thrown upward with an initial speed of 35 m/s, how long does it take to reach the highest point of its path?

Solution. Here you are directly given only $v_0 = 35$ m/s, and you are asked for a time. Indirectly, you are given $v = 0$ because the time requested is to the top of the path where the ball is at rest at the instant when it reverses its direction of motion. Solve Equation (3) for t, and substitute in the values:

$$t = (v - v_0)/(-9.8\ m/s^2)$$

$$= (v_0 - v)/9.8\ m/s^2$$

$$t = (35\ m/s)/(9.8\ m/s^2)$$

$$t = 3.6\ s$$

Problem 6. A single-acting pile driver hammer falls for 0.3 s. What is its speed at the end of the fall?

Problem 7. A ball is thrown downward off a cliff at 20 m/s. What is its speed after 2 s?

Problem 8. A ball is thrown upward with an initial speed of 65 m/s. How long does it take to reach the top of its path? How long does it take before it is traveling downward at 65 m/s?

DISTANCE TRAVELED UNDER CONSTANT ACCELERATION

In many free-fall and other constant-acceleration problems, we wish to know the relation between distance traveled and elapsed time. One formula for distance comes from the definition of average speed, Equation (1).

$$d = v_{av}t \tag{4}$$

However, this equation is not very useful for a uniformly accelerated object, because usually we do not know the average speed. For this reason we use a graph of speed versus time under constant acceleration to get a formula for average speed which can be used with Equation (4). Figure 10 shows such a graph for an object with an initial speed v_0 and a final speed v at time t.

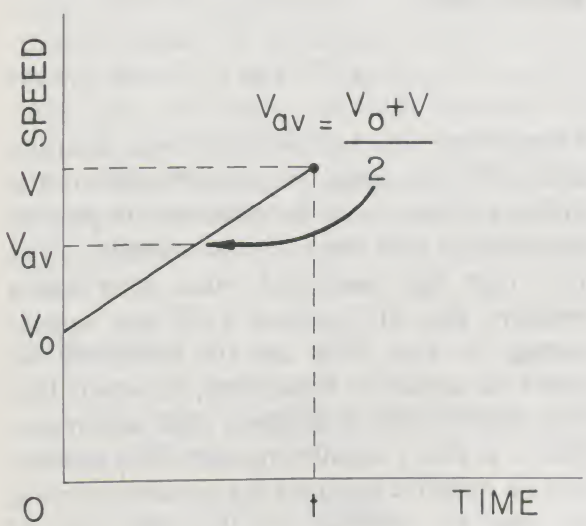


Figure 10.

As you can see, the speed increases linearly (in a straight line) from v_0 to v during time t. The question is, what is the average speed describing this motion? From the graph, it appears that the speed halfway between v_0 and v should be taken as the average value. Half the time the object is moving slower than that and half the time it is moving faster. Every slower speed is matched by a corresponding speed that is just as much faster. Another way of saying the same thing is to say that the average speed of an object with a constant acceleration is the average of the initial speed and final speed. This average is obtained by adding the two values and dividing by 2:

$$v_{av} = (v_0 + v)/2 \tag{5}$$

(Average Speed under Uniform Acceleration)

We now combine Equation (5) with Equation (4) to find the distance traveled:

$$d = [(v_0 + v)/2]\,t \tag{6}$$

We also use the formula for final speed which can be obtained from Equation (2), and which is similar to Equation (3):

$$v = v_0 + at \tag{7}$$

Here we are using the fact that, for a uniform (constant) acceleration, the acceleration at any instant is the same as the average acceleration.

Substituting (7) into (6) gives

$$d = v_0 t + \tfrac{1}{2} at^2 \tag{8}$$

(Distance Traveled under Uniform Acceleration)

Problem 9. Starting with Equation (2), show how Equation (7) is obtained. Then using Equation (7) and Equation (6), show all the steps needed to get Equation (8).

The special form of this equation for a freely falling object uses

$$a = g = -9.8 \text{ m/s}^2$$

and

$$d = v_0 t - 4.9\,t^2 \qquad (9)$$

(Distance Fallen in Free Fall)

Because the value of g is somewhat different at different locations, we can write a more general equation as

$$d = v_0 t + \tfrac{1}{2} g t^2$$

where the numerical value of g is negative.

Example Problem. A rock is dropped down a well and takes 3 s to produce a splash. How deep is the well?

Solution. Because the rock is dropped rather than thrown, you know $v_0 = 0$. Also $t = 3$ s and $g = -9.8$ m/s.

$$d = v_0 t + \tfrac{1}{2} g t^2$$

$$= 0 + \tfrac{1}{2} \times (-9.8 \text{ m/s}^2) \times 9 \text{ s}^2$$

$$d = -44.1 \text{ m}$$

The minus sign indicates a distance downward; the well is therefore 44.1 m deep.

Problem 10. A ball is thrown upward with an initial speed of 65 m/s. How high does it go?

Problem 11. If a single-acting pile driver hammer falls for 0.5 s, how far does it travel?

Problem 12. In Experiment A-1, you dropped a metal weight from 100 cm onto a nail. How long did the weight take to make the trip?

Problem 13. From a standing start, a dragster travels 400 m in 6 s. What is its acceleration?

SPEED AND DISTANCE UNDER CONSTANT ACCELERATION

Problems 8 and 10 illustrate one way to find the distance traveled by an object in free fall, knowing only its initial and final speeds. You discovered another relation between final speed and distance in Experiment B-1: final speed squared is proportional to distance traveled when $v_0 = 0$. Using d for distance fallen:

$$v^2 \propto d$$

This result is consistent with our previous calculations. To show this, we eliminate t from Equation (8) by substituting the value of t from Equation (7). Since we are applying the formulas primarily to pile drivers, we can simplify by setting $v_0 = 0$ in both (7) and (8), so that

$$v = at \qquad (10)$$

and

$$d = \tfrac{1}{2} a t^2 \qquad (11)$$

From (10) we have

$$t = v/a \qquad (12)$$

We substitute (12) into (11) to obtain

$$d = \frac{1}{2} \cdot a \cdot \frac{v^2}{a^2} = \frac{1}{2} \cdot \frac{v^2}{a} = \frac{v^2}{2a}$$

or

$$v^2 = 2\,ad \qquad (13)$$

For free fall

$$v^2 = 2\,gd \qquad (14)$$

Thus, if an object is dropped from rest, the square of its speed is proportional to the distance fallen, and the constant of proportionality is just twice the acceleration. The fact that the numerical value of g has a negative sign in Equation (14) may appear strange to you. How can the square of the speed be negative? Remember, however, that any object which is *dropped* falls downward; thus d is also a negative number. The product of two negative numbers is a positive number, so that the product on the right side of Equation (14) *is* positive. When using Equation (14) it may be easier to choose down-

ward as positive; then both quantities are positive. The right side of Equation (14) is *always* positive if g and d are in the same direction.

Example Problem. A pile driver hammer is dropped from 3 m. Find the speed with which it strikes the pile.

Solution. You are given $d = 3$ m. Substituting into Equation (14):

$$v^2 = 2gd = 19.6 \text{ m/s} \times 3 \text{ m}$$

$$v^2 = 58.8 \text{ m}^2/\text{s}^2$$

therefore

$$v = \sqrt{58.8 \text{ m}^2/\text{s}^2} = 7.67 \text{ m/s}$$

Problem 14. With what speed did the hammer strike the pile in Experiment A-1 when it fell 100 cm? 200 cm? From what height must the hammer be released in order to attain a speed of 98 m/s just before impact? Does this height depend on the mass of the hammer?

ACCELERATION OF GRAVITY AT DIFFERENT LOCATIONS

The value $g = 9.8 \text{ m/s}^2$ is approximately correct only on the surface of the earth. If you could perform Experiment B-1 far above the earth's surface, you would find a smaller value for the acceleration of gravity. On the moon's surface the acceleration due to gravity is only about 1.6 m/s^2. In general, the acceleration due to gravity depends on where in space it is measured. Even on the surface of the earth, the value varies slightly from location to location. For our purposes, however, such variations are too small to be important.

SUMMARY

You have seen that the definition of average speed is

$$v_{av} = \Delta d/\Delta t$$

where Δd is distance traveled, and Δt is the time required. The average speed over a very short time interval is called the instantaneous speed.

You have also learned the mathematical definition of acceleration as the time rate of change of speed. With v_0 as initial speed, v as final speed, and t as the time required for the speed to change from one value to the other, a uniform acceleration a is

$$a = (v - v_0)/t$$

You discovered that objects acted upon only by gravity move with a constant acceleration, $g = -9.8 \text{ m/s}^2$, where the minus indicates that the acceleration is directed downward.

Further, the distance traveled by an object under constant acceleration a is given by

$$d = v_0 t + \tfrac{1}{2} a t^2$$

In pile driving, the set depends on the free-fall distance and on the mass of the hammer. In this section you found that the square of the speed of a dropped object is proportional to the free-fall distance. Every different free-fall distance produces a different hammer speed at impact. Therefore we think of the set as depending directly on the speed of the hammer and only indirectly on the free-fall distance. The formula for the speed of a dropped object is

$$v^2 = 2gd$$

In our discussion of speeds and accelerations so far we have considered *only* those kinds of motion occurring along straight-line paths. In such cases, the words "speed" and "velocity" have the same meaning. However, if you were to consider motion along curved paths, you would find that acceleration can occur when the speed is constant. The velocity of an object can be described adequately only if its instantaneous direction is *also* part of that description. Further, changes in direction alone, or in combination with speed changes, can result in the object having an acceleration. To treat this kind of motion, we need to use *vectors.* For this module, we will restrict our coverage to straight-line motion.

GOALS FOR SECTION C

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal.

1. *Goal:* Understand the definition and concept of mass.

   Item: An electron repels another charged particle. During the interaction it is noted that the other particle's acceleration is 4.26×10^{-3} times the electron's acceleration. What is the mass of the particle compared to the mass of the electron?

2. *Goal:* Know the definitions of force and weight.

   Item: What is the weight of a 2-kg mass on the surface of the moon where the acceleration of gravity is one-sixth that on the earth?

3. *Goal:* Know the mathematical definitions of work, kinetic energy, and gravitational potential energy.

   Item: A 2200-kg ram of a drop hammer has a free-fall distance of 3 m. What is its kinetic energy just before it strikes the pile?

4. *Goal:* Understand the principle of conservation of energy.

   Item: A single-acting steam hammer has a 3500-kg ram with a 1-m free-fall distance. If it transfers two-thirds of its energy to driving the pile per blow and produces a set of 3 cm, what average force does it exert on the pile?

5. *Goal:* Be able to apply the principle of conservation of momentum.

   Item: An automobile having a mass of 1000 kg and moving forward at 7 m/s is hit from behind by a truck moving at 25 m/s. The truck has a mass of 5000 kg and it sticks to the car after the collision. How fast are the two moving at that time?

Answers to Items Accompanying Previous Goals

1. 235 times the electronic mass
2. 3.27 N
3. 6.47×10^4 J
4. 7.62×10^5 N
5. 22 m/s

SECTION C

DEFINITION OF MASS

In Section B of this module, you investigated the speed of an object, such as a pile driver hammer, which is falling. You found that the initial height of the drop hammer, and thus its final speed, affects the set of the pile. The other property important in determining the set is the weight of the drop hammer. In this section we shall consider this second factor more carefully.

In scientific language, a distinction is made between *weight* and the closely related concept of *mass.* The mass of an object does not change as the object is moved from one place to another, but its weight may. For example, the weight of an object on the surface of the planet Jupiter would be greater than on earth, but its mass is the same on Jupiter as on earth. We describe the mass of an object as a measure of the property of the object that resists any change in the object's motion (*acceleration*). This property is called *inertia.* Such a description is not good enough for scientific use because it does not tell us how to measure the mass.

To define a quantity such as mass, scientists use an *operational definition.* In an operational definition, an experiment, or set of operations, is described and the quantity of interest is defined as the result of some measurement made during the experiment. To define mass, for example, we start by choosing a standard unit of mass. That is, we define the mass of a particular object to be one unit of mass. In the metric system this unit of mass is called a *kilogram.* The kilogram is defined as the mass of a particular platinum-iridium cylinder which is kept at the International Bureau of Weights and Measures near Paris. To define the mass of any other object, we do an experiment in which this new object interacts in some way with the standard unit mass. Two objects are said to *interact* if the presence of one object affects the motion of the other object. One example is two masses connected by a stretched spring or rubber band, as shown in Figure 11. When the two

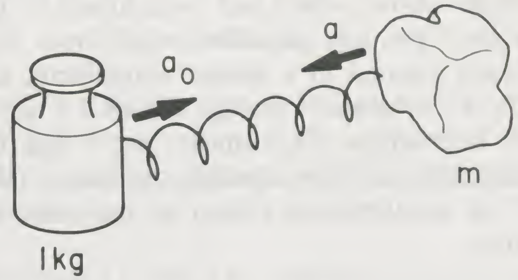


Figure 11.

masses are released and allowed to move, they accelerate toward each other. If one acceleration is negative, the other must be positive. Suppose we measure the two accelerations, finding a_0 for the unit (kilogram) mass and a for the unknown mass, m. Then the unknown mass (in kilograms) is defined as the negative of the ratio of the two accelerations. Since the accelerations have opposite signs, the negative sign assures that the resulting mass is positive.

$$m = -a_0/a \qquad (15)$$

For example, if $a_0 = -6\ \text{m/s}^2$ and $a = 3\ \text{m/s}^2$, then

$$m = -[(-6\ \text{m/s}^2)/(3\ \text{m/s}^2)] = +2 \text{ units}$$

Many experiments of this type show that the ratio is always constant for the same two masses regardless of the kind of interaction or where in space the masses happen to be.

Of course, the International Bureau does not hand out the standard kilogram for such experiments; other methods for determining mass exist. But these other methods are valid because they are logically consistent with the definition just given.

Note that Equation (15) says that the mass of an object is inversely proportional to the acceleration it receives in an interaction with the standard mass. This result is consistent with our word description of mass as resistance to acceleration: large masses receive small accelerations and vice-versa.

INTERACTIONS WITH OTHER MASSES

Suppose that we had two different objects whose masses had been found by the method we just described. Let those two objects interact in a similar experiment, say with a compressed spring. Figure 12 shows this interaction. Experiments show that the ratio of the masses is equal to the inverse ratio of the accelerations (taken at the same instant).

$$m_1/m_2 = -a_2/a_1 \qquad (16)$$

This result is consistent with Equation (15), which could be written:

$$m/1 = -a_0/a$$

We conclude that the mass of an object is a constant property which gives a measure of the object's resistance to acceleration in any interaction.

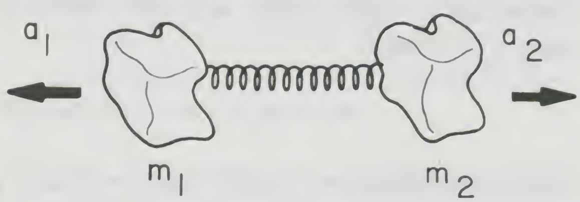


Figure 12.

Example Problem. Two magnets, one with an unknown mass and the other with a mass of 0.8 kg, are held near each other on a nearly frictionless surface so that they strongly repel each other. Just after they are released, the 0.8-kg magnet has an acceleration of -2 m/s^2 and the acceleration of the other magnet is 3.2 m/s^2. What is the mass of the second magnet?

Solution. Given are $m_1 = 0.8$ kg, $a_1 = -2$ m/s^2, and $a_2 = 3.2$ m/s^2. Which mass is chosen as m_1 is immaterial. Since the actual form of interaction is not important, the masses do not have to be connected. The mass can also be measured by magnetic, electric, or gravitational interactions. We can solve Equation (16) for m_2 and substitute into it the known values:

$$m_2 = -\frac{a_1}{a_2}\, m_1$$

$$= -[(-2 \text{ m/s}^2)/(3.2 \text{ m/s}^2)](0.8 \text{ kg})$$

$$= 0.5 \text{ kg}$$

The unknown mass has a value of 0.5 kg.

Problem 15. A metal block is connected to a 1-kg mass by a compressed spring. When the two are released, the metal block has an acceleration of 2 m/s^2 and the 1-kg mass has an acceleration of 5 m/s^2. What is the mass of the metal block?

Problem 16. A 1000-kg hammer of a pile driver is held away from the earth by the rig. If the earth has a mass of 6×10^{24} kg, what is the earth's acceleration toward the hammer when the hammer is released? (Recall that the hammer has an acceleration $g = 9.8$ m/s^2).

Question 4. Does Equation (16) follow logically from the definition of mass? That is, can (16) be derived from (15) without using the results of any new experiments?

DEFINITION OF FORCE

In this module we have spoken of weight and of interactions due to springs, magnets, and gravity. All of these interactions are examples of *forces.* However, we have not yet given a precise definition of force although we have even used the word "forceful." Since force is a very important concept in all of physics and its applications including pile driving, we shall define it here. In descriptive terms a force is thought of as a push or a pull on something. You have some intuitive feeling for what force is because you can feel a push or a pull if it is large enough. This description is not a scientific definition of force because it gives no clear indication of how to measure it or how to assign a numerical value to it.

Once mass is understood, force can be defined by a simple equation. This states that the force F on an object is the object's mass m times its acceleration a.

$$F = ma \qquad (17)$$
(Definition of Force)

With the units we have been using, mass in kg and acceleration in m/s^2, the units of force are $kg \cdot m/s^2$. The definition embodied in Equation (17) may seem arbitrary, but many experiments have shown that the mass times the acceleration of an object corresponds to the push or pull on it. This is indicated in Figure 13.

Equation (17) is known as *Newton's second law of motion,* because Sir Isaac Newton (1642-1727) was the first person to define our intuitive notion of force as a quantity equal to mass times acceleration. Newton's laws of motion represented a great advance in science; the metric (SI) unit of force has been named the *newton* (N) in his honor:

$$1\ N = 1\ kg \cdot m/s^2$$

A newton of force is about 1/4 pound.

Example Problem. What was the force acting on the 0.8-kg mass of the previous example problem?

Solution. You were given $m_1 = 0.8$ kg and $a_1 = -2\ m/s^2$. These values can be substituted directly into (17).

$$F = ma = (0.8\ kg)(-2\ m/s^2)$$

$$F = -1.6\ N$$

The negative sign tells you the direction of the force.

Problem 17. What was the force acting on the 1-kg mass of Problem 15 at the instant its acceleration was measured? What was the force acting on the other mass?

Problem 18. What force acts on a 200-g mass when it is in free fall?

WEIGHT AS A FORCE

Now we can distinguish between the weight and the mass of an object. The *weight* of any object is defined as the force of gravity on it. We are certain that an object in free fall has a force on it because we can measure its acceleration. Furthermore, the force is constant because the acceleration is constant. If we use w for weight and g for the gravitational acceleration, we have from Equation (17)

$$w = mg \qquad (18)$$

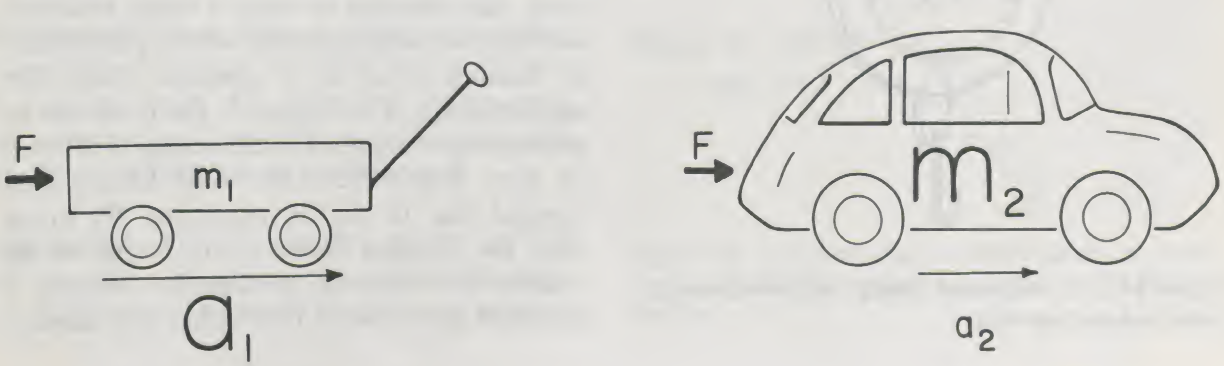


Figure 13. The same force acting on different masses produces different accelerations.

Even in space where the acceleration of gravity g is not equal to $-9.8\ m/s^2$, the weight is given by the mass times the acceleration of gravity.

Because weight is a force, it should be measured in newtons rather than mass units. However, when you weigh objects in the lab, your results are in grams or kilograms. Since weight and mass are always proportional, balances can be calibrated either in weight units or in mass units. The balance can be thought of as comparing either the weight of an unknown with standard weights or the mass of the unknown with standard masses.

Some instruments measure force correctly anywhere if they are correctly calibrated somewhere, and other instruments do the same for mass. For example, a spring balance measures the stretching or compression of a spring, which in turn depends on the force exerted on the spring. An equal-arm balance compares an unknown mass with standard masses, and thus measures mass correctly no matter what the gravitational attraction on the masses (so long as there is *some* attraction). See Figure 14A and B.

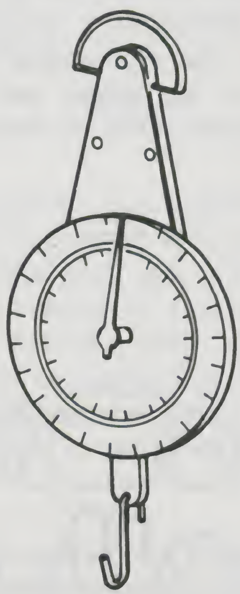


Figure 14A. A calibrated spring balance correctly measures force anywhere.

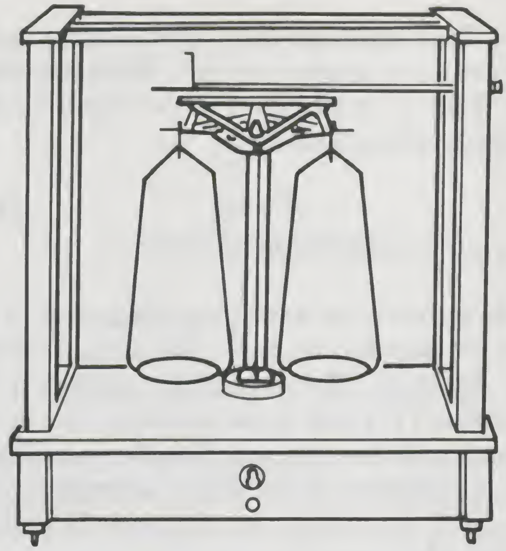


Figure 14B. A calibrated equal-arm balance correctly measures mass anywhere.

Problem 19. What is the weight of a 70-g mass at the earth's surface?

Problem 20. If a 5-kg mass and a 2-kg mass are transported to a planet where the 2-kg mass weighs 30 N, what is the weight of the 5-kg mass?

MULTIPLE FORCES ACTING ON AN OBJECT

According to Equation (17), only one value of force acts on an object because the object can have only one acceleration at a given time. However, this *net force* on an object may be the result of several pushes or pulls. That is, several forces with different causes may act on an object all at the same time, but combine to cause a single resultant acceleration. For example, when a parachutist is floating down at a constant speed (no acceleration), as in Figure 15, the force due to gravity (his weight w) is still acting downward on him. But there is an equal force acting upward due to the air resistance. We know that the upward force is the same as his weight, but opposite in direction because a constant speed means that he has zero acceler-

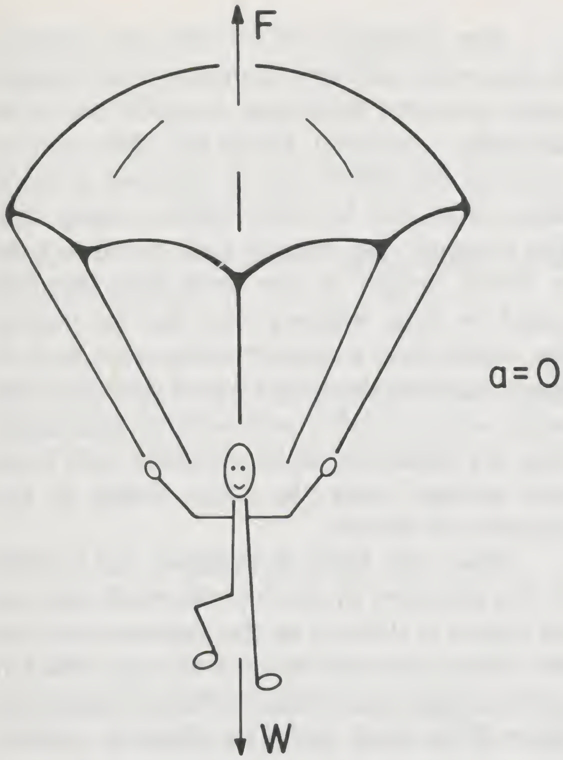


Figure 15. The net acceleration of the parachute is zero because the upward force of air resistance cancels the weight of the man.

ation. From Equation (17), this means that the net force is also zero. Thus the two forces, his weight and the upward push of air, are equal but opposite.

Another way of looking at the same situation is to note that, if air resistance were absent, the force of gravity would cause an acceleration equal to $-9.8\ m/s^2$. Also, if somehow there were a force equal to that of air resistance acting upward by itself, it would cause an upward acceleration equal to $+9.8\ m/s^2$.

Problem 21. As it falls, an 850-kg pile driver hammer is subject to a 300-N frictional force due to the leads. The frictional force acts in an upward direction. What is the net force on the hammer? What is its acceleration?

Question 5. You are standing on the surface of the earth. What force is acting downward on you? How big is it? Is there an upward force acting on you, and if so, what is its size? Give the reasoning behind your answer.

NEWTON'S THIRD LAW OF MOTION

One result that can be deduced from our previous discussion of mass and force concerns interacting objects. If two objects interact, through a compressed spring or in any other manner, Equation (16) states that

$$m_1/m_2 = -a_2/a_1$$

Multiply both sides of this equation by the product m_2a_1 to obtain

$$m_1a_1 = -m_2a_2$$

But m_1a_1 is the force on object 1 due to object 2 and $-m_2a_2$ is the force on object 2 due to object 1, so that

$$F_1 = -F_2 \qquad (19)$$

The conclusion is that whenever two objects interact, the two forces (one exerted on each object by the other) are equal and opposite in direction. This conclusion is known as *Newton's third law of motion,* because it was first explicitly stated by him at the same time as his second law. One example is shown in Figure 16.

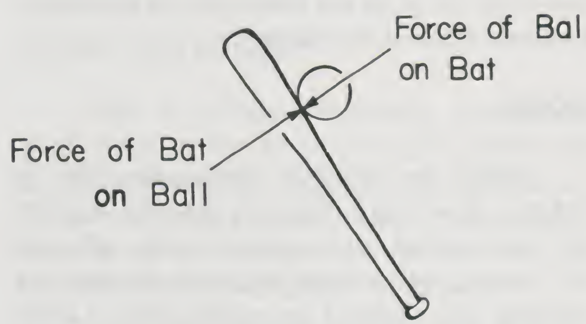


Figure 16. The force exerted by the bat on the ball is equal and opposite to the force exerted on the ball by the bat.

The two forces in Equation (19) are often labeled *action* and *reaction,* although which is called action and which reaction is arbitrary. Newton's third law may then be stated: *for every action there is an equal and opposite reaction.* It is most important to remember that the action and reaction forces in Newton's third law are forces which act on *different* objects. They *never* act on the same object!

Question 6. What is wrong with the following analysis of a motion problem? A horse pulls on a cart. But the cart pulls back with an equal and opposite reaction. Therefore, the horse cannot make the cart move.

Question 7. When the hammer of a pile driver is released, it falls under the force of gravity, its weight. What is the reaction to this force; what object is it acting on?

Question 8. When the pile driver hammer strikes the pile, it exerts a large force downward on the pile. What is the reaction to this force and what object is it acting on?

NEWTON'S FIRST LAW OF MOTION

After seeing Newton's second and third laws of motion, you may by now be wondering what happened to the first law. The first law is in actuality a special case of the second: when the net force acting on an object is zero, the acceleration of the object is zero. That is, if no *net* force acts on an object, its speed remains the same.

WORK

When the hammer strikes the pile, it exerts a downward force on the pile. Analyzing the detailed forces acting on the pile and its consequent motions is a complicated and difficult task. Instead, we must resort to some simplifying concepts used by scientists and engineers to analyze such complicated situations. One of these concepts is called *work.*

The scientist's use of this word "work" is somewhat different from and considerably more restricted than your everyday use of it. Basically, a scientist would say that *work* is done on an object only if whatever is being done could *not* be done without using fuel. For example, you require fuel (food) to hold a 300-lb weight at one level. But that job could be done without using fuel by placing the weight onto a support at the same level. It turns out that the only kind of definition for work which satisfies such a restrictive condition is a definition which includes both force and motion, with the force acting in the direction of motion.

When the force is constant, and parallel to the direction of motion, the work done on an object is defined as the force exerted on the object times the distance through which it moves while the force is acting. Using F for force, W for work, and d for distance, we have

$$W = Fd \tag{20}$$

As indicated in Figure 17, if the same force acts through the same distance on different objects, the work done is the same.

Even in cases like a pile driver where the force is *not* constant, we can still use this definition if we use the *average* value of the force.

The units for work are newtons times meters, which are given the name *joules* (J):

$$1\ \text{N}\cdot\text{m} = 1\ \text{J}$$

Whenever a force is exerted on an object and it moves in the direction of the force, we say that work is done on it. However, if the object does not move, no work is done on it regardless of the forces acting.

Example Problem. A pile driver hammer strikes the pile, exerting an average downward force of 10^4 N. If the set of the pile is 2 cm, what work does the hammer do on the pile?

Solution. Given are $F = 10^4$ N and $d = 2 \times$

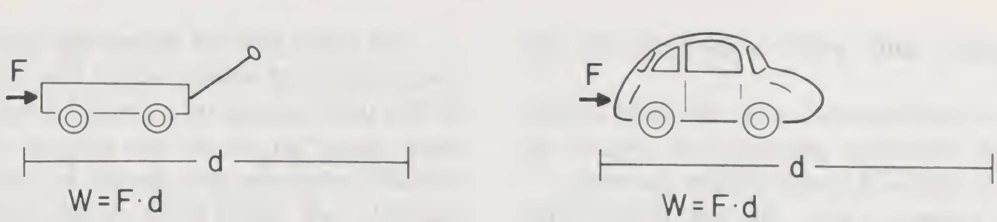


Figure 17. The same force F acting through the same distance d does the same work W, even though it produces different accelerations.

10^{-2} m, which can be substituted into Equation (19):

$$W = 10^4 \text{ N} \times 2 \times 10^{-2} \text{ m}$$

$$= 2 \times 10^2 \text{ N·m}$$

$$W = 200 \text{ J}$$

Problem 22. If a pile driver hammer does 1000 J of work on a pile whose set is 3 cm, what is the average force it exerts on the pile?

WORK DONE BY THE FORCE OF GRAVITY

When the pile driver hammer falls, the force of gravity does work on it. The constant force exerted by gravity is the weight of the hammer mg and the distance moved is the free-fall distance h. Thus the work done on the hammer by gravity is given by

$$W = mgh \qquad (21)$$

Since both g and h have been defined as negative quantities, the work W is positive. Saying this somewhat differently, the force and the motion are both in the same direction, downward, so the work is positive.

The same amount of work must be done to lift the hammer back up to a height h. Neglecting friction, the upward force needed is very nearly equal to the weight of the hammer (but opposite in direction). This force is exerted through a distance equal to h (but opposite in direction).

You have seen that for a given free-fall distance h, the hammer has a certain speed v, just before hitting the pile. In fact, that speed is given by

$$v^2 = 2gh$$

or

$$gh = \tfrac{1}{2} v^2 \qquad (22)$$

Since the work done on the hammer by gravity as the hammer falls is given by

$$W = mgh$$

we can use the value of gh given in Equation (22) to show that the work is also given by

$$W = m\,(gh) = m\,(\tfrac{1}{2} v^2)$$

or

$$W = \tfrac{1}{2} mv^2$$

WORK AND ENERGY

This is a very interesting result. The work done by the force of gravity on the pile driver hammer as it falls through a distance h is mgh. But this amount of work has the same value as $\frac{1}{2} mv^2$, where v is the speed of the hammer after it has fallen a distance h.

When the hammer is lifted to a height h prior to releasing it, fuel of some kind must be used. In the sense that fuel consumption "uses energy," we have used energy in raising

the hammer and work was done in the process.

In a more general way, we may define *energy* as whatever property an object or system has which makes it able to do work.

For example, when the pile driver hammer is raised to a height h, the *system* of earth plus the hammer stores energy in the form of gravitational *potential energy.* This system can do work by exerting a force on the hammer (its weight mg) as it falls through a distance h.

In general, potential energy is the energy a body or system has because of its *position.* For example, a stretched spring stores potential energy because of the position of its ends.

Returning to the pile driver hammer, when it is released from its raised position, it loses its potential energy as it falls. Just before it hits the pile, it has lost the potential energy it acquired by being lifted. However, it is still able to do work; it exerts a force—a large one—on the pile and moves it through a distance, the set. Yet, if you were to just place the hammer on the pile, nothing much would happen. Therefore, we conclude that the hammer's energy must be in its motion just before impact. This "energy of motion" is called *kinetic energy* and is given by $\frac{1}{2} mv^2$. More precisely, if m is the mass of an object and v is its speed, its kinetic energy is:

$$KE = \tfrac{1}{2} mv^2 \qquad (23)$$

Problem 23. Show that the units of kinetic energy are joules.

CONSERVATION OF ENERGY

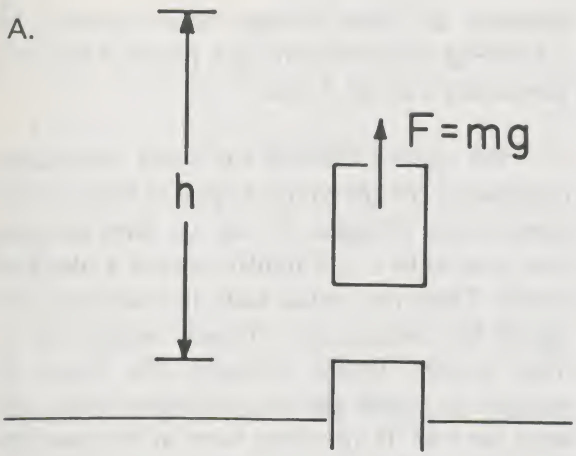
Figure 18 summarizes the ideas of the preceding discussion. In that discussion we have implicitly used one of the most powerful laws of physics. When we assume that the stored potential energy mgh is converted into kinetic energy $\frac{1}{2} mv^2$ as the hammer falls, we are assuming that energy is *conserved,* or kept constant. When work was done on the hammer to raise it, the energy was stored as potential energy.

In more general terms, the principle of *conservation of energy* states that the *change* in the total energy of a system is equal to the work done on or by the system. A *system* includes whatever you *decide* to include. For example, we could think of the earth and a pile driver hammer as a system. For a system to have work done on it, or to do work, there must be yet another system external to it. For the earth and hammer as a system, the pile driver lifting apparatus and engine can form another system. When the pile driver hammer is raised (separated further from the earth) by the lifting apparatus and engine, work is done on the earth-and-hammer system by the lifting aparatus-and-engine system. The result of that work is to increase the potential energy of the earth-hammer system by mgh. When the hammer falls, the lifting apparatus-and-engine system is no longer doing any work on the earth-and-hammer system. However, by conservation of energy, the total energy of the earth-and-hammer system must remain constant, if no *other* system does work on it. When the hammer is at the raised position and at rest, it has no kinetic energy. Therefore its total energy is just its potential energy at that height. As the hammer falls, its potential energy gets smaller, while its kinetic energy increases. But the total energy, kinetic plus potential, remains the same until just before the hammer hits the pile. At that time, the decrease in potential energy (mgh) has been transformed to kinetic energy ($\frac{1}{2} mv^2$), and

$$mgh = \tfrac{1}{2} mv^2$$

When the hammer hits the pile, it drives the pile into the ground a distance equal to the set. It does work which is *internal* to the earth-and-hammer system. If we want to analyze this situation, we must consider the pile as a *new* system, and the earth and the hammer as *other* systems. Then we can consider the work done on the pile by the forces acting on the pile as it is driven into the ground. The hammer exerts forces on the pile and the earth exerts forces on the pile. These forces are not constant, nor are they always

A.



B.

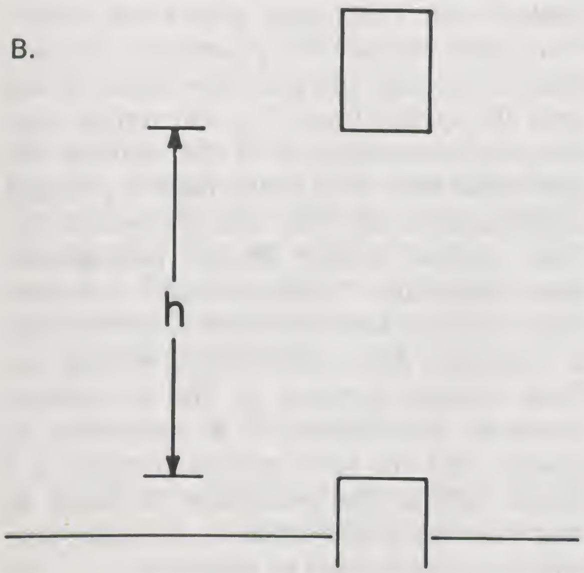
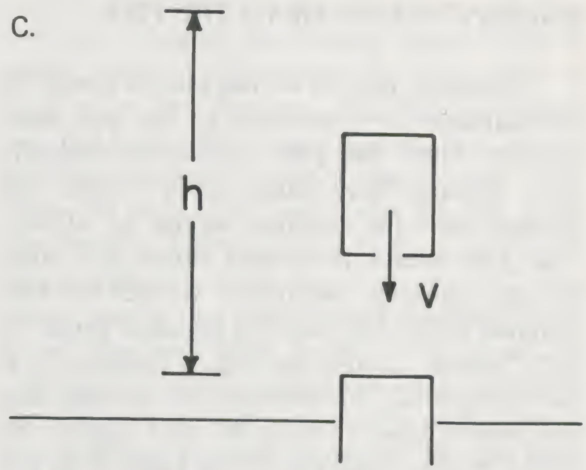


Figure 18A. Work is put into the pile driver hammer in raising it to a height h.

Figure 18B. When the hammer is suspended at height h, it stores potential energy mgh.

C.



D.

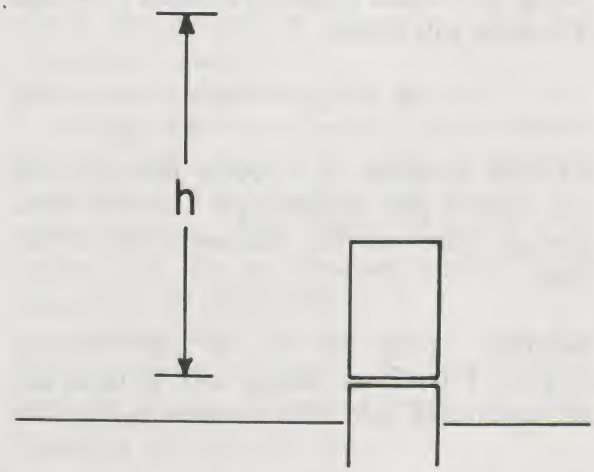


Figure 18C. When the hammer falls, it loses potential energy and gains kinetic energy. Just before impact $\frac{1}{2}mv^2 = mgh$.

Figure 18D. Just before impact $\frac{1}{2}mv^2 = mgh$.

equal. The energy transformations in this interaction are complicated and involve yet another form of energy, *heat*. Still, the total energy of the earth-pile system is conserved.

As hinted at above, another statement of the conservation of energy is that the total energy of an isolated system is constant.

Problem 24. How much work is done by steam pressure to lift an 800-kg hammer 1.5 m in a pile driver? How much potential energy does the hammer have when lifted into position? Suppose the hammer is released. How much kinetic energy does the hammer have at the instant before it hits the pile? How fast is the hammer moving at that instant?

ENERGY TRANSFER TO THE PILE

Suppose that all of the kinetic energy of the hammer is transferred to the pile upon impact. Then the pile would, immediately after impact, have some speed v into the ground, and the hammer would be at rest. The pile would then slow down and stop, having moved a distance s, the set for that hammer blow. But then the hammer would be left "sitting" above the pile a distance s. It would then fall that distance, hit the pile, and eventually come to rest. We will neglect the effect of this small fall of the hammer in our analysis of the problem.

Suppose the hammer transfers all of its kinetic energy to the pile. It does an amount of work on the pile equal to mgh, the change in the potential energy of the hammer as it falls. The work done on the pile by the hammer is given by the average force F_{av} acting on the pile times the distance s through which the pile moves.

$$W = F_{av} \cdot s = mgh \tag{24}$$

Example Problem. A 1000-kg hammer falls 2 m onto a pile, producing a 2-cm set. What average force did the hammer exert on the pile?

Solution. Given are $m = 10^3$ kg, $h = 2$ m, and $s = 2 \times 10^{-2}$ m. Along with g, these can be substituted into (24) solved for F_{av}.

$F_{av} = mgh/s$

$= (9.8 \text{ m/s}^2 \times 10^3 \text{ kg} \times 2 \text{ m})/2 \times 10^{-2} \text{ m}$

$F_{av} = 9.8 \times 10^5 \text{ kg}\cdot\text{m/s}^2 = 9.8 \times 10^5 \text{ N}$

You may recall that a 1-N force is about one-fourth pound. Also, a 1000-kg mass weighs about 2200 lb. Thus, this 2200-lb hammer exerts an average force of about 245,000 lb during the blow.

Problem 25. If a 200-g mass falls 20 cm onto a nail to produce a set of 1.2 mm, what average force does it exert?

Problem 26. What average force is exerted by a 1500-kg hammer striking a pile at 6 m/s and producing a set of 5 cm?

We need to search for some reasonable expression for the average applied force as the penetration changes. To do so, first suppose that you hold a nail lightly against a block of wood. Then you could start the nail into the wood by placing just enough weight on it. This weight would produce the force f_0 needed to push the wood fibers aside and start the nail. If you then were to increase this force by adding weight to the nail, the nail would be pushed into the wood a little bit farther each time more weight was added. You could measure the penetration for each load on the nail, and graph the resulting data with the applied force F on the vertical axis, and the penetration P of the nail on the horizontal axis. This is not really a practical experiment to do. This is partly because the force required to start the nail moving each time, called the "breaking force," is greater than the force needed to keep it moving once it is started. Also, after it starts moving, the force needed increases as the penetration increases. Nonetheless, it is reasonable to assume that the force needed increases in a simple way as the penetration increases. In that case the graph would be a straight line, and its equation could be written as

$$F = f_0 + cP \tag{25}$$

where c is the slope of the line, and f_0 is the "breaking force." Figure 19 is a graph of this equation.

In this equation the product cP is the difference between the applied force F and the force f_0 needed to push the wood aside. We identify it as equal in magnitude to the *frictional force* between the nail and the wood.

Assuming that Equation (25) might also have some approximate validity for a pile, we would like to use it to find the work done on a pile with each blow of the hammer. With a set s the work done on the pile by the hammer each time is:

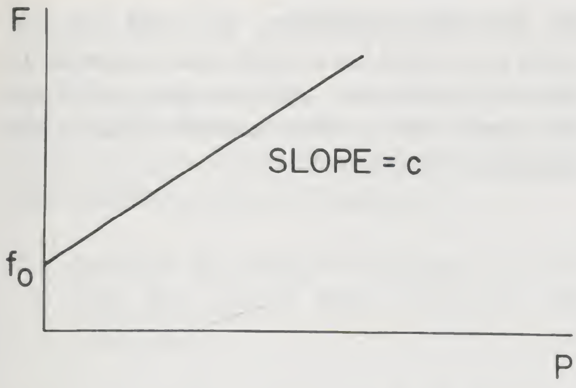


Figure 19. Graph of applied force versus penetration of a nail.

$$W = F_{av}s$$

We must use an average force here because, as the pile moves through a distance s, the force changes slightly. We will assume that s is small enough that the force changes only a little and that Equation (25) can be used as a reasonable approximation to F_{av}.

In that case we get:

$$W \approx (f_0 + cP)s$$

If we assume that all of the energy of the hammer, mgh, is transferred to the pile, we get:

$$(f_0 + cP)s = mgh \qquad (26)$$

We will analyze this equation under two different assumptions and see which one makes sense. First, assume that the initial penetration force f_0 is small compared with the frictional force, cP. Under this assumption, after the first blow, the penetration is just the set, s_1. The force F during the second blow has the approximate value $f_0 + cs_1$. After the second blow, the penetration is $s_1 + s_2$, and the force is approximately $c(s_1 + s_2)$. After some later blow, say the tenth, the penetration is the sum of the first ten values of set, $(s_1 + s_2 + \ldots s_{10})$, and the force of the next blow is $c(s_1 + s_2 + \ldots s_{10})$. (However if f_0 is very much smaller than cP, we can ignore it, and the force is approximately cP.) The first few drops of the hammer produce forces which increase from blow to blow. This is because the set *decreases*, and the smaller the set, the larger the force required. But, as the pile penetration becomes large, each new value of set adds very little to the total penetration. That is, $(s_1 + s_2 + \ldots s_{10})$ is not much different from $(s_1 + s_2 + \ldots s_{11})$. In other words, the force stays essentially the same from one blow to the next. Thus, for penetrations which are much larger than the set, the average force exerted by each drop of the hammer is nearly constant. We can then write

$$Ks \approx mgh \qquad (27)$$

(Large Penetration)

where K is a constant force. Therefore, we predict that the set s after a large penetration is directly proportional to the height from which the hammer is dropped *if* this first assumption is correct.

Equation (26) and the assumption that f_0 is much less than cP imply that the average force on the pile is quite small for the first few blows. If the hammer falls through the same distance each time, Equation (26) implies that the set becomes smaller as the penetration becomes greater.

Alternatively, we can assume that the force f_0 required to start the pile into the ground is large compared with cP. Then Equation (26) becomes just

$$f_0 s \approx mgh \qquad (28)$$

Under this assumption, the set depends only on the height from which the hammer is dropped. The set is proportional to the height for *all* values of penetration, not just for large penetrations as before.

Which assumption correctly describes a nail being driven into a block of wood by a falling hammer? Does the set start out large and decrease to a constant value for large penetration? Or does the set remain essentially the same for each blow of the hammer?

Equation (27) and Equation (28) *both* imply that the set does *not* depend on the mass of the nail, even though we don't know which (if either) of these assumptions is correct for a hammer driving a nail into wood. To test this implication, you will use two nails, one which has a small mass compared to that of the hammer, and the other which has the same mass as the hammer. This is the purpose of Experiment C-1.

EXPERIMENT C-1. Energy Transfer in Pile Driving

The purpose of this experiment is to test the validity of the equation $(f_0 + cP)s = mgh$. To do this you will use two different nails and two weights to serve as "hammers."

1. Measure the mass m and length L_1 of a nail and record these values in the worksheet.

To hold the nail in place, put it into one of the holes which have been drilled into the wooden block. The hole has a diameter slightly larger than the nail. Using a hammer start the nail into the wood (0.5 cm or so).

Let x_i* be the distance from the top of the nail to the surface of the wooden block, as shown in Figure 20.

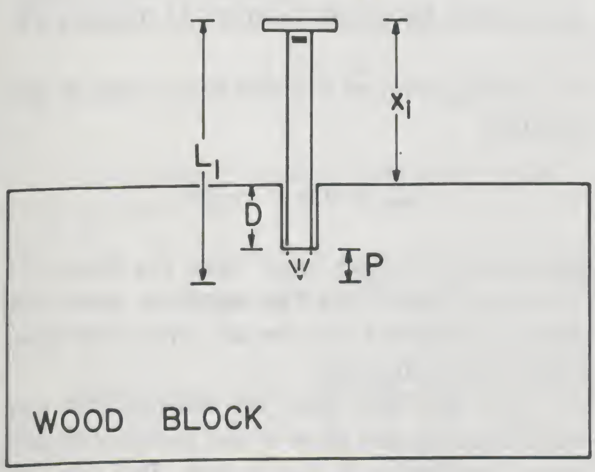


Figure 20.

The length L_1 of the nail has the same value as the sum of the distance x_i, the depth of the support hole D, and the penetration P. Thus,

$$L_1 = x_i + D + P$$

or

$$P = L_1 - D - x_i$$

*x_i is the distance between the top of the nail and the block before the i^{th} blow from the hammer. Thus x_1 is the initial distance, x_2 the distance before the second blow, etc.

For two consecutive values of x_i, say x_2 and x_3, the set is just their difference, $s = x_3 - x_2$.

2. Determine the set s and penetration P for eight consecutive blows of the hammer (weight) dropped from a height of 100 cm. Measure the mass of the hammer and record its value.

3. From the measurements and calculations you made in steps 1 and 2, which assumption concerning the force terms in Equation (26) do you believe is more nearly correct? Is the force term f_0 much larger than the term cP, or vice versa?

4. Using the values of hammer mass m_H, hammer height h, and g (9.8 m/s^2), calculate the value of the potential energy, $m_H gh$. (Be sure to express m_H in kilograms and h in meters.)

You probably found that the set is essentially constant for a nail being driven into wood by a falling weight. Therefore, the second assumption, f_0 is much greater than cP, is more nearly correct. The average value of the set for the eight trials can be used to find the average force exerted on the nail during each blow. That is,

$$F_{av} s = m_H gh$$

so that

$$F_{av} = \frac{m_H gh}{s}$$

5. Divide the value of $m_H gh$ calculated in step 4 by the average set s_{av}. What is the result? (Be sure to express s_{av} in meters.)

You now know the average force which the weight exerts on the nail. Now change the mass of the nail. Changing the mass of the nail does *not* change the value of the potential

energy of the weight being dropped. Since

$$F_{av}s = m_H gh$$

is assumed to be true for any nail, we should get exactly the same result with another nail of different mass. We will not change the size of the nail itself, or its sharpness. We will use the same wood. The values of f_0 and c in Equation (26) are determined by the size and sharpness of the nail and by the properties of the wood into which the nail is being driven. According to our theory and assumptions, the force exerted on the nail by the wood as the nail is driven should be exactly as it was before. In turn, the average set s_{av} should be the same. You will find out if it is.

Take the nail you just used and push it through the hole in the metal cylinder provided, as shown in Figure 21. Then lock the cylinder onto the nail with the set screws. Enough of the nail should protrude through the bottom of the cylinder to provide for the same total penetration you had before.

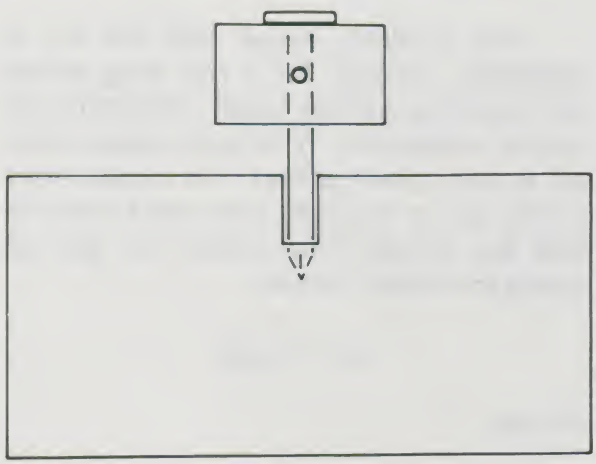


Figure 21.

6. Repeat steps 1 through 5 for this "heavier" nail. Actually this arrangement has been selected to be equivalent to a nail of the same mass as the weight you are dropping on it. Space has been left in the worksheet for these new measurements and calculations.

According to our theory, the average force F_{av} depends only on the height h, the hammer mass m_H, the size and sharpness of the nail, and the properties of the wood. None of these variables has changed from one nail to the next. Do you calculate the same force?

7. What do you conclude about the validity of Equation (26)?

ANALYSIS OF EXPERIMENTAL RESULTS

In Experiment C-1 you found that in the equation

$$(f_0 + cP)s = m_H gh$$

the force f_0 is much larger than the force cP. You also found that this equation does not correctly account for the set when the mass of the nail is changed.

For the first nail, we assume that the work done by one blow of the hammer at any given penetration is $f_0 s = mgh$. But for the more massive nail, the set is smaller at the same penetration and $f_0 s$ is *less* than mgh. Since mgh is still the energy available, some energy has been lost—it was not converted into work.

Where did this missing energy go?

MOMENTUM

The blow of the hammer on the pile is an example of a collision. As is illustrated by the difficulty just pointed out, the concept of energy is not a complete tool for analyzing such problems. A second concept is needed to permit a more complete description of collisions. That concept is called *momentum.* Using momentum *and* energy, the collision of the hammer and pile can be analyzed more correctly.

By our earlier definition, force is given by

$$F = ma \tag{17}$$

If F is an average force, the acceleration a is an average value. You may recall that average acceleration is given by

$$a_{av} = \frac{v - v_0}{t} \tag{29}$$

Combining Equation (29) and Equation (17), we have

$$F_{av} = m\frac{v - v_0}{t}$$

Multiplying both sides of this equation by t, we have

$$F_{av}t = mv - mv_0 \tag{30}$$

The product of the average force F_{av} and the time t during which it acts, is called the *impulse*:

$$\text{Impulse} = F_{av}t \tag{31}$$

The product of the mass and the speed is called the *momentum**:

$$\text{Momentum} = mv \tag{32}$$

We would like to apply Equation (30) separately to the hammer and the pile. But before doing this, we should describe what actually happens when a pile is driven. Figure 22 shows a graph of the speed of a pile versus elapsed time for one blow of the pile driver hammer.

You can see that, from the instant the hammer hits the pile at $t = 0$, until time $t = t_1$, the speed quickly increases from zero to a maximum, v_p. During this time the pile is accelerating. If the pile is accelerating, a net force must be acting on the pile. During the time from t_1 until t_2, the speed decreases, but at a much lower rate than it originally increased. This negative acceleration is numerically small compared with that during the time from $t = 0$ to $t = t_1$. After time t_2 the speed *oscillates* back and forth, positively and negatively, finally decreasing to zero. We can explain this behavior as follows.

*In two- or three-dimensional motion, the *direction* of the momentum is also of importance.

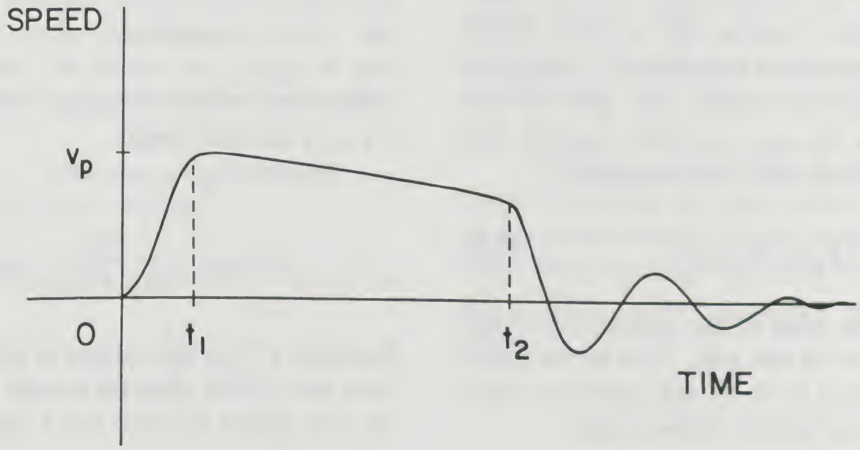


Figure 22.

The time between zero and t_1 is the time during which the hammer hits the pile. In this very short time, the hammer loses kinetic energy and the pile acquires its maximum kinetic energy. When this interaction ends at time t_1, the hammer and pile are moving into the ground at the same speed with the hammer riding on the pile. Between time t_1 and shortly after time t_2, the kinetic energy of the pile decreases to zero. The remaining kinetic energy of the hammer also decreases to zero. Both the pile and the hammer lose a small amount of potential energy as they move together through a distance equal to the set s.

The behavior of the pile after t_2 is extremely complex. As the pile is driven, both the pile itself and the ground into which it is being driven are compressed. This is similar to compressing a spring. At time t_2 these compression forces become very large; the pile stops quickly, and even moves in the opposite direction. The pile and hammer bounce rapidly back and forth, but these oscillations quickly die out. The vibrations continue until the potential energy stored in the compression has been "used up." We will see later where the energy goes.

During the short time of impact from $t = 0$ to $t = t_1$, the pile and hammer move a distance which is very small compared to the set s.

Now let us apply Equation (30) for the collision. Consider the pile first. The total force acting on the pile is the average force of the hammer acting on the pile F_p plus the weight of the pile itself $m_p g$. As you have seen, the force exerted by a pile driver hammer during impact is enormous compared with the weight of either the pile or the hammer. Thus we can properly neglect this weight, and we can write the equation,

$$F_p t_1 \cong m_p v_p \tag{33}$$

where m_p is the mass of the pile and v_p is the maximum speed of the pile. This is the speed of the pile after it is hit by the hammer, when it has not yet had time to move much.

Now consider the average force of the pile acting on the hammer F_H. The speed of the hammer immediately *before* the collision is v_H. *After* the collision the hammer and pile are moving at the *same* speed, the maximum speed of the pile v_p. Thus the momentum of the hammer changes by

$$m_H v_p - m_H v_H$$

For this collision, we can write Equation (30) for the hammer as

$$F_H t_1 = m_H v_p - m_H v_H \tag{34}$$

The force on the hammer due to the pile is, by Newton's third law, equal and opposite to the force on the pile due to the hammer.

$$F_H = -F_p$$

Then Equation (34) becomes

$$-F_p t_1 = m_H v_p - m_H v_H$$

If we add this equation to Equation (33) (since the time t_1 of the collision is the same for both), we have

$$0 = m_p v_p + m_H v_p - m_H v_H$$

Rearranging this equation, we have

$$v_p(m_p + m_H) = m_H v_H$$

As indicated in Figure 23, this equation says that the momentum after the collision $(m_p + m_H)v_p$ is equal to the momentum before the collision $m_H v_H$. Such an equality is a very general result.

Solving for v_p, we have

$$v_p = \frac{m_H}{m_p + m_H} v_H \tag{35}$$

Problem 27. A pile driver is driving piles in a river bed. Each pile has a mass of 400 kg and the pile driver hammer has a mass of 1000 kg.

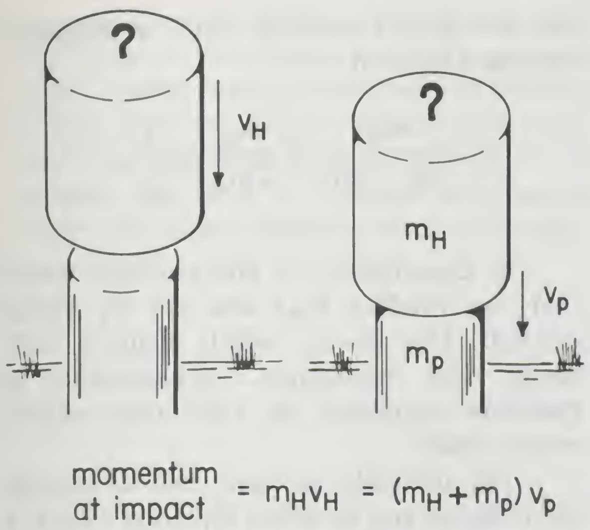


Figure 23. The momentum before impact of the pile driver hammer is equal to the momentum after impact.

If the hammer is dropped from a height of 3 m, how fast are the pile and hammer moving together immediately after the hammer hits the top of the pile?

The speed v_p is the speed which *both* the hammer and the pile have as they start to move through the distance *s* of the set. The kinetic energy the two objects have at this speed is converted into the work of driving the pile the distance *s*. We can calculate this kinetic energy. For the pile, it is

$$\tfrac{1}{2} m_p v_p^2$$

For the hammer moving at the same speed, it is

$$\tfrac{1}{2} m_H v_p^2$$

Adding these two energies, we have the total kinetic energy

$$\tfrac{1}{2} (m_p + m_H) v_p^2$$

But from Equation (35), we can write v_p^2 as

$$v_p^2 = \left(\frac{m_H}{m_p + m_H}\right)^2 v_H^2$$

Thus, we have for the total kinetic energy (KE_T) of the pile and the hammer right after collision,

$$KE_T = \frac{1}{2}(m_p + m_H) \frac{m_H^2}{(m_p + m_H)^2} v_H^2$$

This simplifies to

$$KE_T = \frac{1}{2} \frac{m_H^2}{(m_p + m_H)} v_H^2$$

This can be written in a slightly different form as

$$KE_T = \left(\frac{m_H}{m_p + m_H}\right) \frac{1}{2} m_H v_H^2 \qquad (36)$$

Notice that the factor $\tfrac{1}{2} m_H v_H^2$ is the total kinetic energy the system had just before impact when the hammer had all the energy. Since $(m_p + m_H)$ is greater than m_H, the total kinetic energy after impact is less than $\tfrac{1}{2} m_H v_H^2$. This supports your observation in Experiment C-1 that energy appears to be lost. In fact, the preceding discussion indicates that energy is apparently lost in the collision but momentum remains the same (is *conserved*).

Problem 28. How much potential energy does the hammer described in Problem 27 have before it is dropped? How much kinetic energy does the hammer have immediately before hitting the pile? How much kinetic energy do the pile and hammer together have immediately after the hammer hits the pile?

We have claimed that some of the work done in driving the pile comes from the loss of potential energy of the hammer and pile as they drop through the distance equal to the set *s*. This potential energy would have the value

$$(m_p + m_H) g s$$

A detailed analysis of this energy for nails and for pile drivers at large penetrations shows it to be very much smaller than the kinetic energy of Equation (38), so it can be ignored. Problem 29 will give you an idea of how much smaller it is for a particular case.

Problem 29. Refer to Problems 27 and 28. Suppose that for a hammer drop height of 3 m the set is 0.05 m. How much potential energy of the hammer and pile is converted into work as the pile is driven the distance equal to the set? How does this amount of energy compare with the kinetic energy of the hammer and pile together which you calculated in Problem 28?

Since the pile and hammer come to rest together after moving a distance s (and bouncing a little), essentially all of the kinetic energy has been used to do the work $F_{av}s$,

$$F_{av}s = KE_T$$

Also,

$$\tfrac{1}{2} m_H v_H^2 = m_H g h$$

Combining these with Equation (36) gives

$$F_{av}s \approx \frac{m_H}{m_p + m_H} m_H g h$$

Finally, the force F_{av} can be written as

$$F_{av} = f_0 + cP$$

where f_0 is the force necessary to push the wood aside, and cP is the force required for penetration P. Then we have

$$(f_0 + cP)s = \frac{m_H}{m_p + m_H} m_H g h \qquad (37)$$

The result is the same as Equation (26), except for the factor $m_H/(m_p + m_H)$ on the right. In Experiment C-1, you used a nail which had the same mass as the hammer. It appeared that some energy was lost for that nail. For such a case the factor in Equation (36) has a value of

$$\frac{m_H}{m_H + m_H} = \frac{m_H}{2\, m_H} = \frac{1}{2}$$

In Experiment C-1 you probably found that the product $F_{av}s$ was low by about one-half. Our theory, which includes both energy and momentum considerations, is therefore supported by your own experimental results.

The principle we have used to analyze the collision and to derive Equation (37) is a special example of a very general principle called the *Law of Conservation of Momentum.* This powerful generalization applies to every kind of known interaction: gravitational, electrical, magnetic, even nuclear. The law can be stated very simply:

> The total momentum of a system after an interaction is equal to the total momentum before the interaction, provided no external forces act on the system.

This law is also approximately correct if the interaction force is much larger than the external forces during the time of the interaction.

In order to correctly describe a collision, *both* the conservation of energy and the conservation of momentum may be applicable. Momentum is *always* conserved in collisions. Whether or not *mechanical energy** is conserved depends on the nature of the collisions. In fact, one can classify collisions on the basis of whether or not mechanical energy is conserved. If mechanical energy *is* conserved, the collision is said to be *elastic.* If mechanical energy is *not* conserved, the collision is *inelastic*. The collision of a pile driver hammer with a pile is inelastic.

Even though we have correctly explained how the pile is driven, we have *not* yet

*Mechanical energy is the sum of kinetic and potential energies.

accounted for the energy apparently lost. For both nails of Experiment C-1, what happens to the kinetic energy of the nail and hammer as they drive the nail into the wood and then stop? The kinetic energy has vanished. There certainly has been no increase in potential energy of the nail (indeed, there has been a slight decrease). A harder question to answer is the following: why is less of the potential energy of the hammer transferred to the pile for a heavier nail than for a lighter nail, and what has happened to this energy? Experiment C-2 will help you answer these questions.

EXPERIMENT C-2. Temperature of the Pile

In this experiment you will observe the temperature change of a pile as it is driven. To approximate a pile, you will use a nail with a *thermistor* attached. A thermistor is a device whose electrical resistance decreases as its temperature increases. This property is used to measure temperatures.

Drive the nail halfway into the board. Position the nail under the large weight.

Connect the leads of the thermistor to the amplifier-power supply. A change in the meter reading is roughly proportional to the change in temperature of the thermistor.

By now, the nail should be near room temperature. Touch the head of the nail with your finger. The meter reading should increase as the nail is heated by your finger. When the temperature of the nail starts to rise, remove your finger. Observe the meter until it returns to its initial position. If the meter reading does not drop, readjust the gain control to produce a zero reading.

1. Record the meter reading. Then drop the large weight onto the nail from a height of 100 cm.

2. Does the meter reading change? Is the direction of the change in meter reading the same as when you held your finger to the nail?

3. Does the temperature of the nail increase or decrease?

4. Can you explain what causes the change in temperature?

5. Can you predict what would happen to the temperature if you dropped the small weight from the same height? (That is, would the meter reading change by a larger, smaller, or the same amount?) Drop the smaller weight and test your prediction.

SUMMARY OF EXPERIMENT C-2

The temperature of the nail increases slightly as it is driven into the wood. Why? Suppose you are driving a car and have to make a "panic stop." You apply the brakes, lock the wheels and skid to a stop. What has happened to the kinetic energy of the car? Its potential energy has not increased. However, an inspection of the tires would indicate that they are hot–much hotter than before the stop. The presence of "skid marks" on the road is additional evidence of an increase in the temperature of the tires and the road surface. The rubber has been bonded to the road by high temperatures. The kinetic energy of the car has disappeared, and a temperature increase is observed. The kinetic energy of a falling weight disappears and a temperature increase is observed. Have you ever seen a bullet which has been fired into or against a stone surface? It is just a melted blob. When a falling weight drives a nail, the kinetic energy of the weight and nail disappear and the temperature of the nail is observed to increase. These observations imply, but do *not* prove, that kinetic energy has been transformed into a form of energy which causes a temperature change. We call this energy form *thermal energy*, or *heat*.

The idea of heat as an energy form allows further generalization of the law of energy conservation. Heat is another form into which energy may be transformed by doing work. In the case of the pile driver, the work done by the frictional forces of the earth on the pile results in a transformation of hammer kinetic energy to heat energy. This answers the first question about the missing energy. The second question has a similar answer. The temperature of the hammer and pile increase due to their own deformities, which are inelastic. They thereby get warmer due to this energy "loss" as well. We will not pursue this idea further, but you may gain further insight into the nature of heat by studying the Physics of Technology modules on *The Pressure Cooker* and *The Power Transistor*.

SUMMARY

You have learned that mass is defined by its acceleration in an interaction with the standard unit of mass. If, as a result of an interaction, the mass m receives an acceleration a, and the standard mass receives an acceleration a_0, then m is defined by

$$m = -a_0/a$$

If any two masses interact, they accelerate in opposite directions and the ratio of their masses is equal to the negative of the inverse ratio of the accelerations

$$m_1/m_2 = -a_2/a_1$$

This equation is one way of expressing Newton's third law of motion.

Force is defined as mass times acceleration:

$$F = ma$$

which is Newton's second law of motion.

If the net force is zero, then no acceleration occurs. That is, a force is necessary to cause acceleration. This special case of Newton's second law is called Newton's first law.

If a number of causes act to accelerate an object, we say that a number of forces act; each force is equal to the mass times the acceleration that would be produced by that cause alone.

Two forces in the same direction add to give a total force. Two forces in opposite directions must be subtracted.

Weight is a force given by the mass of an object times the acceleration of gravity (mg).

Work is defined as a force times the distance through which it is exerted, provided the force is exerted in a direction parallel to the distance.

$$W = Fd$$

The work done by gravity when an object falls through a height h is given by

$$W = mgh$$

Energy is the ability to do work. Two important kinds of energy for pile drivers are gravitational potential energy mgh and kinetic energy $\frac{1}{2}mv^2$. The amounts of energy, given by these expressions, indicate how much work an object can do. Energy is always *conserved*: it is neither created nor destroyed in any interaction, although it can change from one form to another.

Momentum is also conserved. The total momentum of a system after an interaction is equal to the total momentum before the interaction, provided no external forces act during the interaction.

In the pile driver, the potential energy of the hammer is transformed into kinetic energy as the hammer falls. During the collision of the hammer with the pile, most of the momentum of the hammer is transferred to the pile. However, not all of the kinetic energy of the hammer is transferred to the pile. Some is transferred into heat due to inelastic deformities of the hammer and pile. While the pile is being driven, the acquired kinetic energy of the hammer and pile plus a small amount of potential energy given up by the hammer and the pile are also converted into heat. The heat produced raises the temperature of the pile and the surrounding ground.

Heat can be considered as a form of energy. In collisions, other forms of energy are often converted to heat.

WORKSHEETS

EXPERIMENT A-1. Driving a Pile with a Falling Weight

Name____________________________

1. Results of weighing nails

   m_1 = __________

   m_2 = __________

   m_3 = __________

   Small weight: M_S = __________

   Large weight: M_L = __________

2. L_1 = __________cm

3. D = __________cm

4.-10. Results for nail 1

Table 1.
Small Weight at Height of 200 cm

Length of Nail above Wood D (cm)	Set (cm)	Penetration P (cm)

11. L_2 = __________cm

12. Results for nail 2

Table 2.
Small Weight at Height of 100 cm

Length of Nail above Wood (cm)	Set (cm)	Penetration (cm)

13. L_3 = __________cm

14. Results for nail 3

Table 3.
Large Weight at Height of 100 cm

Length of Nail above Wood (cm)	Set (cm)	Penetration (cm)

15. Graph set vs. penetration

16. ______________________________

17. ______________________________

18. ______________________________

19. ______________________________

WORKSHEETS

EXPERIMENT B-1. Motion of a Freely Falling Body

Name____________________________________

3.-6. Results of measurements

Table 4.

Time $t = 0$	Distance between Marks (Δd)	Total Distance Fallen (d)	Average Speed ($v_{av} = \Delta d/\Delta t$)	Average Acceleration ($a_{av} = \Delta v_{av}/\Delta t$)
t_1				
t_2				
t_3				
t_4				
t_5				
t_6				
t_7				
t_8				
t_9				
t_{10}				

7. ______________________________

8. Average of accelerations ________________

9. Graph of v_{av} versus d

10. ______________________________

11. Other graphs

12. ______________________________

13. ______________________________

14. ______________________________

15. ______________________________

WORKSHEET

EXPERIMENT C-1. Energy Transfer in Pile Driving

Name____________________________________

1. First nail

   $h = 100$ cm

   $m =$ ____________ g

   $L_1 =$ ____________ cm

2. Depth of support hole

   $D =$ ____________ cm

Drop Number	Top of Nail to Wood Distance x_i	Set s $x_i - x_{i-1}$	Penetration P $L_1 - D - x_i$
0	$x_0 =$ ________		
1	$x_1 =$ ________		
2	$x_2 =$ ________		
3	$x_3 =$ ________		
4	$x_4 =$ ________		
5	$x_5 =$ ________		
6	$x_6 =$ ________		
7	$x_7 =$ ________		
8	$x_8 =$ ________		

Mass of hammer $m_H =$ __________ g

3. ____________________________________

   ____

4. $m_H gh =$ ____________ J

5. $F_{av} = \dfrac{m_H gh}{s_{av}} =$ ____________ =

   ____________ N

6. "Heavier" nail

   $h = 100$ cm

   $m =$ __________________ g

   $L_1 =$ __________________ cm

   Depth of support hole:

   $D =$ __________________ cm

Drop Number	Top of Nail to Wood Distance x_i	Set s $x_i - x_{i-1}$	Penetration P $L_1 - D - x_i$
0	x_0 = ______		
1	x_1 = ______		
2	x_2 = ______		
3	x_3 = ______		
4	x_4 = ______		
5	x_5 = ______		
6	x_6 = ______		
7	x_7 = ______		
8	x_8 = ______		

Mass of hammer m_H = __________g

3. ______________________________

____

4. $m_H gh$ = ______________J

5. $F_{av} = \frac{m_H gh}{s}$ = ______________

= ______________N

7. ______________________________

____

____

COMPUTATION SHEET

WORKSHEETS

EXPERIMENT C-2. Temperature of the Pile

Name________________________________

1. Initial meter reading ______________

2. Meter reading after fall _____________

   Change in meter reading ____________

3. Direction of change of temperature (higher or lower) _____________

4. Explanation of this change

5. Prediction:

   Was your prediction correct?

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